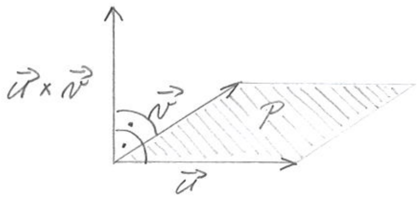
VEKTOROVÝ SOUČIN VEKTORŮ

$\vec{u} \times \vec{v} =$
- $\vec{o}$ ... jestliže $\vec{u}, \vec{v}$ jsou kolineární nebo $\vec{u} = \vec{o}$ nebo $\vec{v} = \vec{o}$
- $\vec{u} \times \vec{v}$ ... vektor kolmý k $\vec{u}$ i $\vec{v}$

$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \sin\varphi$, $\varphi \in \langle 0, \pi \rangle$
- obsah rovnoběžníku určeného vektory $\vec{u}, \vec{v}$



Platí: 1) $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

2) $\alpha \cdot (\vec{u} \times \vec{v}) = (\alpha \cdot \vec{u}) \times \vec{v} = \vec{u} \times (\alpha \cdot \vec{v})$

3) $(\vec{u} + \vec{v}) \times \vec{w} = \vec{u} \times \vec{w} + \vec{v} \times \vec{w}$

4) $\vec{w} \times (\vec{u} + \vec{v}) = \vec{w} \times \vec{u} + \vec{w} \times \vec{v}$

Využití:
- Kolineárnost vektorů ... $\vec{u} \times \vec{v} = \vec{o} \Leftrightarrow \vec{u}, \vec{v}$ kolineární
- Výpočet obsahu rovnoběžníku / trojúhelníku
- Nalezení vektoru kolmého ke dvěma daným nenulovým vektorům

V souřadnicích: $\vec{a} = (a_1, a_2, a_3)$, $\vec{b} = (b_1, b_2, b_3)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$= \vec{i} \cdot (-1)^{1+1} \cdot \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + \vec{j} \cdot (-1)^{1+2} \cdot \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \vec{k} \cdot (-1)^{1+3} \cdot \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} =$$

$$= \vec{i}(a_2 b_3 - a_3 b_2) - \vec{j}(a_1 b_3 - a_3 b_1) + \vec{k}(a_1 b_2 - a_2 b_1)$$

Př: Jsou dány vektory $\vec{a}=(1,2,1)$, $\vec{b}=(2,-1,3)$.
a) Zjistěte, zda $\vec{a}, \vec{b}$ jsou kolineární,
b) Vypočtěte obsah rovnoběžníku nad vektory $\vec{a}, \vec{b}$,
c) Určete vektor $\vec{x}$ kolmý k $\vec{a}, \vec{b}$ délky 15.

a) $\vec{a}, \vec{b}$ kolineární $\Rightarrow \vec{a}\times\vec{b}=\vec{o}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 2 & 1\\ 2 & -1 & 3\end{vmatrix}=\vec{i}\cdot\begin{vmatrix}2 & 1\\ -1 & 3\end{vmatrix}-\vec{j}\cdot\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix}+\vec{k}\cdot\begin{vmatrix}1 & 2\\ 2 & -1\end{vmatrix}=$$

$$=\vec{i}\cdot(6+1)-\vec{j}\cdot(3-2)+\vec{k}\cdot(-1-4)=7\vec{i}-\vec{j}-5\vec{k}=$$

$$=(7,-1,-5)\neq\vec{o} \Rightarrow \underline{\underline{\vec{a}, \vec{b} \text{ nejsou kolineární}}}$$

b) $S=\|\vec{a}\times\vec{b}\|=\sqrt{49+1+25}=\sqrt{75}=\underline{\underline{5\sqrt{3}\ j^2}}$

c) $\vec{x}\perp\vec{a} \wedge \vec{x}\perp\vec{b} \Rightarrow \vec{x}=t\cdot(\vec{a}\times\vec{b})$

$\vec{x}=(7t,-t,-5t)$

$\|\vec{x}\|=15 \Rightarrow \sqrt{49t^2+t^2+25t^2}=5\sqrt{3}\cdot|t|=15$

$\sqrt{3}\cdot|t|=3$

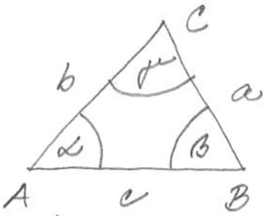
$t=\pm\sqrt{3}$

$t_1=\sqrt{3}\ \dots\ \underline{\underline{\vec{x}_1=(7\sqrt{3},-\sqrt{3},-5\sqrt{3})}}$

$t_2=-\sqrt{3}\ \dots\ \underline{\underline{\vec{x}_2=(-7\sqrt{3},\sqrt{3},5\sqrt{3})}}$

Př: Jsou dány body $A[1,2,0]$, $B[-1,1,3]$, $C[1,3,-1]$.

Určete: a) obsah $\triangle ABC$
b) délky stran $\triangle ABC$
c) délky výšek $\triangle ABC$
d) vnitřní úhly $\triangle ABC$



$\vec{AB} = (-2,-1,3)$ $\quad \vec{BC} = (2,2,-4)$ $\quad \vec{AC} = (0,1,-1)$

$\vec{BA} = (2,1,-3)$ $\quad \vec{CB} = (-2,-2,4)$ $\quad \vec{CA} = (0,-1,1)$

a) $$S = \frac{\|\vec{AB} \times \vec{AC}\|}{2}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 3 \\ 0 & 1 & -1 \end{vmatrix} = \vec{i}(1-3) - \vec{j}(2-0) + \vec{k}(-2-0) = (-2,-2,-2)$$

$$\|\vec{AB} \times \vec{AC}\| = \sqrt{4+4+4} = 2\sqrt{3}$$

$$S = \frac{2\sqrt{3}}{2} = \underline{\underline{\sqrt{3}\ j^2}}$$

b) $$a = \|\vec{BC}\| = \sqrt{4+4+16} = \sqrt{24} = \underline{\underline{2\sqrt{6}}}$$

$$b = \|\vec{AC}\| = \sqrt{0+1+1} = \underline{\underline{\sqrt{2}}}$$

$$c = \|\vec{AB}\| = \sqrt{4+1+9} = \underline{\underline{\sqrt{14}}}$$

c) $$S = \frac{a \cdot v_a}{2} \Rightarrow v_a = \frac{2S}{a} = \frac{2\sqrt{3}}{2\sqrt{6}} = \frac{1}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$v_b = \frac{2S}{b} = \frac{2\sqrt{3}}{\sqrt{2}} = \underline{\underline{\sqrt{6}}}$$

$$v_c = \frac{2S}{c} = \frac{2\sqrt{3}}{\sqrt{14}} = \underline{\underline{\frac{\sqrt{42}}{7}}}$$

d) $$\cos\alpha = \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\| \cdot \|\vec{AC}\|} = \frac{0-1-3}{\sqrt{14} \cdot \sqrt{2}} = -\frac{4}{2\sqrt{7}} = -\frac{2}{\sqrt{7}} \Rightarrow \underline{\underline{\alpha = \arccos\left(-\frac{2}{\sqrt{7}}\right)}}$$

$$\cos\beta = \frac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \cdot \|\vec{BC}\|} = \frac{4+2+12}{\sqrt{14} \cdot 2\sqrt{6}} = \frac{18}{2 \cdot 2\sqrt{7} \cdot \sqrt{3}} = \frac{9}{2\sqrt{7} \cdot \sqrt{3}} = \frac{3\sqrt{3}}{2\sqrt{7}} \Rightarrow$$

$$\Rightarrow \underline{\underline{\beta = \arccos\left(\frac{3\sqrt{3}}{2\sqrt{7}}\right)}}$$

$$\cos\gamma = \frac{\vec{CA} \cdot \vec{CB}}{\|\vec{CA}\| \cdot \|\vec{CB}\|} = \frac{0+2+4}{\sqrt{2} \cdot 2\sqrt{6}} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \underline{\underline{\gamma = \arccos\frac{\sqrt{3}}{2}}}$$