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Lagrangeův a Newtonův interpolační polynom

Teoretický úvod

Lagrangeův interpolační polynom:

$$P(x) = y_0 L_0(x) + \ \dots + y_n L_n(x)$$

Postup výpočtu součinitele L_i(x):

$$L_i(x) = \frac{(x - x_0)(x - x_1) \dots (x - x_{i-1})(x - x_{i+1}) \dots (x - x_n)}{(x_i - x_0)(x_i - x_1) \dots (x_i - x_{i-1})(x_i - x_{i+1}) \dots (x_i - x_n)}$$

Newtonův interpolační polynom:

$$P = y_0 + y[x_0, x_1](x - x_0) + y[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots$$
$$+ y[x_0, \dots, x_n](x - x_0) \dots (x - x_{n-1})$$

Postup výpočtu poměrných diferencí $y[x_0, x_1]$,..., $y[x_0, \dots, x_n]$ je znázorněn v následujícím schématu.

x_i	y_i		$y[x_i, x_{i+1}]$		$y[x_i, \dots, x_{i+n}]$
x_0	y_0				
		$\searrow$			
			$\frac{y_1 - y_0}{x_1 - x_0} = y[x_0, x_1]$		
		$\nearrow$		$\searrow$	
x_1	y_1				$\frac{y_n - y_0}{x_n - x_0} = y[x_0, x_n]$
		$\searrow$		$\nearrow$	
			$\frac{y_n - y_1}{x_n - x_1} = y[x_1, x_n]$		
		$\nearrow$			
x_n	y_n				

Příklad 1:

i	x_i	$y(x_i)$
0	4	3
1	5	2
2	7	0

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-5)(x-7)}{(4-5)(4-7)} = \frac{(x^2-5x-7x+35)}{(-1)(-3)} = \frac{x^2-12x+35}{3}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-4)(x-7)}{(5-4)(5-7)} = \frac{(x^2-4x-7x+28)}{1*(-2)} = \frac{x^2-11x+28}{-2}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-4)(x-5)}{(7-4)(7-5)} = \frac{(x^2-4x-5x+20)}{3*2} = \frac{x^2-9x+20}{6}$$

$$P(x) = 3*L_0(x) + 2*L_1(x) + 0*L_2(x) =$$

$$= 3*\frac{x^2-12x+35}{3} + 2*\frac{x^2-11x+28}{-2} + 0*\frac{x^2-9x+20}{6} =$$

$$= x^2-12x+35-x^2+11x-28+0 =$$

$$= -x+7$$

Newtonův polynom:

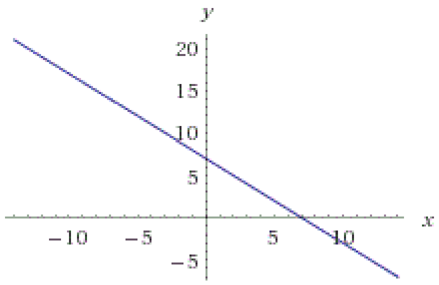
x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
4	3		
		$\frac{2-3}{5-4} = -1$	
5	2		$\frac{-1-(-1)}{7-4} = 0$
		$\frac{0-2}{7-5} = -1$	
7	0		

$$P(x) = y_0 + y[x_0,x_1] * (x - x_0) + y[x_0,x_1,x_2] * (x - x_0)(x - x_1) =$$

$$= 3 + (-1) * (x - 4) + 0 * (x - 4)(x - 5) =$$

$$= 3 - x + 4 =$$

$$= -x + 7$$



Příklad 2:

Na následujícím příkladu si ukážeme, jak náročné může být tyto polynomy vypočítat a jak snadné udělat chybu. Ve složitějších případech je výhodnější použít pouze Newtonův polynom, jehož výpočet je snadnější a rychlejší. Pro kontrolu je však dobré vyšetřit polynom oběma způsoby.

i	x_i	$y(x_i)$
0	-2	1
1	1	5
2	2	0
3	4	2

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-1)(x-2)(x-4)}{(-2-1)(-2-2)(-2-4)} = \frac{(x^2+2x-x+2)(x-4)}{-72}$$
$$= \frac{(x^2+x+2)(x-4)}{-72} = \frac{x^3-4x^2+x^2-4x+2x-8}{-72} = \frac{x^3-3x^2-2x-8}{-72}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x+2)(x-2)(x-4)}{(1+2)(1-2)(1-4)} = \frac{(x^2+2x-2x-4)(x-4)}{9}$$
$$= \frac{(x^2-4)(x-4)}{9} = \frac{x^3-4x^2-4x+16}{9}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x+2)(x-1)(x-4)}{(2+2)(2-1)(2-4)} = \frac{(x^2-x+2x-2)(x-4)}{-8}$$
$$= \frac{(x^2+x-2)(x-4)}{-8} = \frac{x^3-4x^2+x^2-4x+2x+8}{-8} = \frac{x^3-3x^2-2x+8}{-8}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x+2)(x-1)(x-2)}{(4+2)(4-1)(4-2)} = \frac{(x^2-x+2x-2)(x-2)}{36}$$
$$= \frac{(x^2+x-2)(x-2)}{36} = \frac{x^3-2x^2+x^2-2x-2x+4}{36} = \frac{x^3-x^2-4x+4}{36}$$

$$P(x) = 1 * L_0(x) + 5 * L_1(x) + 0 * L_2(x) + 2 * L_3(x) =$$

$$= 1 * \frac{x^3 - 3x^2 - 2x - 8}{-72} + 5\frac{x^3 - 4x^2 - 4x + 16}{9} + 0 + 2 * \frac{x^3 - x^2 - 4x + 4}{36} =$$

$$= \frac{x^3 - 3x^2 - 2x - 8}{-72} + \frac{5x^3 - 20x^2 - 20x + 80}{9} + \frac{2x^3 - 2x^2 - 8x + 8}{36} =$$

$$= \frac{x^3 - 3x^2 - 2x - 8 + 40x^3 - 160x^2 - 160x + 640 + 4x^3 - 4x^2 - 16x + 16}{72} =$$

$$= \frac{45x^3 - 167x^2 - 178x + 648}{72} = \frac{5}{8}x^3 - \frac{167}{72}x^2 - \frac{89}{36}x + 9$$

Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$	$y[x_i,x_{i+1},x_{i+2}, x_{i+3}]$
-2	1 ↘			
		$\frac{5-1}{1+2}=2$ ↘		
1	5 ↗ ↘		$\frac{-5-2}{2+2}=-\frac{7}{4}$ ↘	
		$\frac{0-5}{2-1}=-5$ ↗ ↘		$\frac{2+\frac{7}{4}}{4+2}=\frac{15}{24}$
2	0 ↗ ↘		$\frac{1+5}{4-1}=2$ ↗	
		$\frac{2-0}{4-2}=1$ ↗		
4	2 ↗			

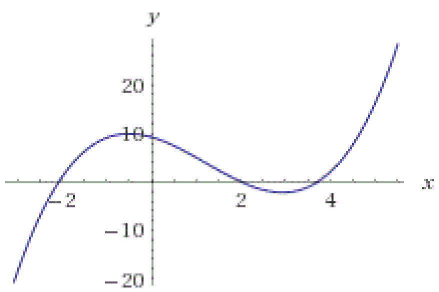
$$P(x) = 0 - 5 * (x-2) - \frac{19}{12} * (x-2)(x-1) + \frac{43}{72} * (x-2)(x-1)(x+2) =$$

$$= -5x + 10 - \frac{19}{12} * (x^2 - 3x + 2) + \frac{43}{72} * (x^2 - 3x + 2)(x+2) =$$

$$= -5x + 10 - \frac{19}{12} * (x^2 - 3x + 2) + \frac{43}{72} * (x^3 - x^2 - 4x + 4) =$$

$$= \frac{-360x + 720 - 114x^2 + 342x - 228 + 43x^3 - 43x^2 - 172x + 172}{72} =$$

$$= \frac{43}{72}x^3 - \frac{157}{72}x^2 - \frac{95}{36}x + \frac{83}{9}$$



Příklad 3:

i	x_i	$y(x_i)$
0	0	2
1	1	6
2	3	0
3	4	-2

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)(0-4)} = \frac{(x^2-3x-x+3)(x-4)}{-12}$$
$$= \frac{(x^2-4x+3)(x-4)}{-12} = \frac{x^3-4x^2-4x^2+16x+3x-12}{-12}$$
$$= \frac{x^3-8x^2+19x-12}{-12}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-0)(x-3)(x-4)}{(1-0)(1-3)(1-4)} = \frac{(x^2-3x)(x-4)}{6}$$
$$= \frac{x^3-4x^2-3x^2+12x}{6} = \frac{x^3-7x^2+12x}{6}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-0)(x-1)(x-4)}{(3-0)(3-1)(3-4)} = \frac{(x^2-x)(x-4)}{-6}$$
$$= \frac{(x^3-4x^2-x^2+4x)}{-6} = \frac{x^3-5x^2+4x}{-6}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-0)(x-1)(x-3)}{(4-0)(4-1)(4-3)} = \frac{(x^2-x)(x-3)}{12}$$
$$= \frac{(x^3-3x^2-x^2+3x)}{12} = \frac{x^3-4x^2+3x}{12}$$

$$P(x) = 2L_0(x) + 6L_1(x) + 0L_2(x) - 2L_3(x) =$$

$$= 2 * \frac{x^3-8x^2+19x-12}{-12} + 6 * \frac{x^3-7x^2+12x}{6} + 0 - 2 * \frac{x^3-4x^2+3x}{12} =$$

$$= -\frac{x^3-8x^2+19x-12}{6} + \frac{6x^3-42x^2+72x}{6} - \frac{x^3-4x^2+3x}{6} =$$

$$= \frac{-x^3+8x^2-19x+12+6x^3-42x^2+72x-x^3+4x^2-3x}{6} =$$

$$= \frac{4x^3-30x^2+50x+12}{6} =$$

$$= \frac{2}{3}x^3 - 5x^2 + \frac{25}{3}x + 2$$

Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$	$y[x_i,x_{i+1},x_{i+2}, x_{i+3}]$
0	2 ↘			
		$\frac{6-2}{1-0}=4$ ↘		
1	6 ↗↘		$\frac{-3-4}{3-0}=-\frac{7}{3}$ ↘	
		$\frac{0-6}{3-1}=-3$ ↗↘		$\frac{\frac{1}{3}+\frac{7}{3}}{4-0}=\frac{2}{3}$
3	0 ↗↘		$\frac{-2+3}{4-1}=\frac{1}{3}$ ↗	
		$\frac{-2-0}{4-3}=-2$ ↗		
4	-2 ↗			

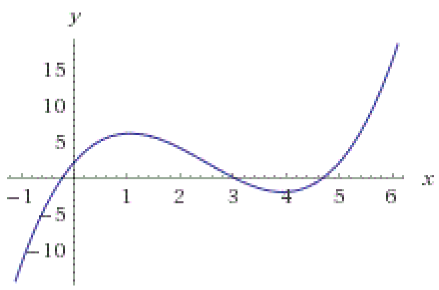
$$P(x) = 2 + 4(x-0) - \frac{7}{3}(x-0)(x-1) + \frac{2}{3}(x-0)(x-1)(x-3) =$$

$$= 2 + 4x + \left(-\frac{7}{3}\right) * (x^2 - x) + \frac{2}{3}(x^2 - x)(x-3) =$$

$$= 2 + 4x - \frac{7}{3}x^2 + \frac{7}{3}x + \frac{2}{3}(x^3 - 3x^2 - x^2 + 3x) =$$

$$= 2 + 4x - \frac{7}{3}x^2 + \frac{7}{3}x + \frac{2}{3}x^3 - \frac{8}{3}x^2 + 2x =$$

$$= \frac{2}{3}x^3 - 5x^2 + \frac{25}{3}x + 2$$



Příklad 4:

i	x_i	$y(x_i)$
0	1	0
1	2	3
2	3	2

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{(x^2-2x-3x+6)}{(-1)*(-2)} = \frac{(x^2-5x+6)}{2}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{(x^2-x-3x+3)}{1*(-1)} = -x^2+4x-3$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{(x^2-2x-x+2)}{2*1} = \frac{(x^2-3x+2)}{2}$$

$$P(x) = 0*L_0(x) + 3*L_1(x) + 2*L_2(x) =$$

$$= 0*\frac{(x^2-5x+6)}{2} + 3*(-x^2+4x-3) + 2*\frac{(x^2-3x+2)}{2} =$$

$$= -3x^2+12x-9+0+x^2-3x+2 =$$

$$= -2x^2+9x-7$$

Newtonův polynom:

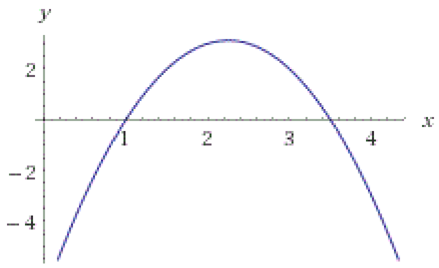
x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
1	0		
		$\frac{3-0}{2-1} = 3$	
2	3		$\frac{-1-3}{3-1} = -2$
		$\frac{2-3}{3-2} = -1$	
3	2		

$$P(x) = y_0 + y[x_0, x_1] * (x - x_0) + y[x_0, x_1, x_2] * (x - x_0)(x - x_1) =$$

$$= 0 + 3 * (x - 1) + (-2) * (x - 1)(x - 2) =$$

$$= 3x - 3 + (-2) * (x^2 - 3x + 2) =$$

$$= -2x^2 + 9x - 7$$



Příklad 5:

i	x_i	$y(x_i)$
0	1	0
1	2	3
2	3	2
3	5	-1

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-2)(x-3)(x-5)}{(1-2)(1-3)(1-5)} = \frac{(x^2-2x-3x+6)(x-5)}{(-1)*(-2)*(-4)}$$
$$= \frac{x^3-5x^2-5x^2+6x+25x-30}{-8} = \frac{x^3-10x^2+31x-30}{-8}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-1)(x-3)(x-5)}{(2-1)(2-3)(2-5)} = \frac{(x^2-x-3x+3)(x-5)}{1*(-1)*(-3)}$$
$$= \frac{x^3-4x^2-5x^2+3x+20x-15}{3} = \frac{x^3-9x^2+23x-15}{3}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-1)(x-2)(x-5)}{(3-1)(3-2)(3-5)} = \frac{(x^2-x-2x+2)(x-5)}{2*1*(-2)}$$
$$= \frac{x^3-3x^2-5x^2+2x+15x-10}{-4} = \frac{x^3-8x^2+17x-10}{-4}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-1)(x-2)(x-3)}{(5-1)(5-2)(5-3)} = \frac{(x^2-x-2x+2)(x-3)}{4*3*2}$$
$$= \frac{x^3-3x^2-3x^2+2x+9x-6}{24} = \frac{x^3-6x^2+11x-6}{24}$$

$$P(x) = 0*L_0(x) + 3*L_1(x) + 2*L_2(x) + (-1)*L_3(x) =$$

$$= 0 + 3*\frac{x^3-9x^2+23x-15}{3} + 2*\frac{x^3-8x^2+17x-10}{-4} + (-1)*\frac{x^3-6x^2+11x-6}{24} =$$

$$= \frac{24x^3-216x^2+552x-360}{24} + \frac{-12x^3+96x^2-204x+120}{24} + \frac{-x^3+6x^2-11x+6}{24} =$$

$$= \frac{11x^3-114x^2+337x-234}{24} =$$

$$= \frac{11}{24}x^3 - \frac{19}{4}x^2 + \frac{337}{24}x - \frac{39}{4}$$

Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$	$y[x_i,x_{i+1},x_{i+2},x_{i+3}]$
1	0			
		$\frac{3-0}{2-1}=3$		
2	3		$\frac{-1-3}{3-1}=-2$	
		$\frac{2-3}{3-2}=-1$		$\frac{-\frac{1}{6}-(-2)}{5-1}=\frac{11}{24}$
3	2		$\frac{-\frac{3}{2}-(-1)}{5-2}=-\frac{1}{6}$	
		$\frac{-1-2}{5-3}=-\frac{3}{2}$		
5	-1			

$$P(x)=y_0+y[x_0,x_1]*(x-x_0)+y[x_0,x_1,x_2]*(x-x_0)(x-x_1)+y[x_0,x_1,x_2,x_3]*$$
$$*(x-x_0)(x-x_1)(x-x_2)=$$

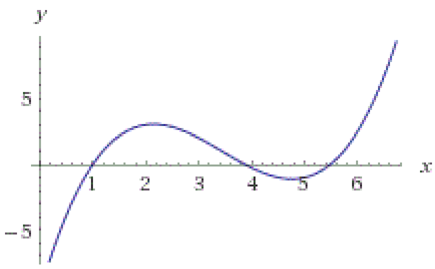
$$=0+3*(x-1)+(-2)*(x-1)(x-2)+\frac{11}{24}(x-1)(x-2)(x-3)=$$

$$=3x-3+(-2)*(x^2-3x+2)+\frac{11}{24}(x^2-3x+2)(x-3)=$$

$$=3x-3-2x^2+6x-4+\frac{11}{24}(x^3-3x^2-3x^2+2x+9x-6)=$$

$$=\frac{72}{24}x-\frac{72}{24}-\frac{48}{24}x^2+\frac{144}{24}x-\frac{96}{24}+\frac{11}{24}x^3-\frac{66}{24}x^2+\frac{121}{24}x-\frac{66}{24}=$$

$$=\frac{11}{24}x^3-\frac{19}{4}x^2+\frac{337}{24}x-\frac{39}{4}$$



Příklad 6:

i	x_i	$y(x_i)$
0	1	2
1	4	4
2	5	1

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-4)(x-5)}{(1-4)(1-5)} = \frac{(x^2-5x-4x+20)}{(-3)*(-4)} = \frac{x^2-9x+20}{12}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-5)}{(4-1)(4-5)} = \frac{(x^2-5x-x+5)}{3*(-1)} = \frac{(x^2-6x+5)}{-3}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-4)}{(5-1)(5-4)} = \frac{(x^2-4x-x+4)}{4*1} = \frac{(x^2-5x+4)}{4}$$

$$P(x) = 2*L_0(x) + 4*L_1(x) + 1*L_2(x) =$$

$$= 2*\frac{x^2-9x+20}{12} + 4*\frac{(x^2-6x+5)}{-3} + 1*\frac{(x^2-5x+4)}{4} =$$

$$= \frac{2x^2-18x+40-16x^2+96x-80+3x^2-15x+12}{12} =$$

$$= -\frac{11}{12}x^2 + \frac{21}{4}x - \frac{7}{3}$$

Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
1	2		
		$\frac{4-2}{4-1}=\frac{2}{3}$	
4	4		$\frac{-3-\frac{2}{3}}{5-1}=-\frac{11}{12}$
		$\frac{1-4}{5-4}=-3$	
5	1		

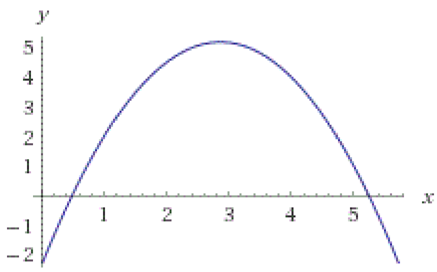
$$P(x) = y_0 + y[x_0, x_1] * (x - x_0) + y[x_0, x_1, x_2] * (x - x_0)(x - x_1) =$$

$$= 2 + \frac{2}{3} * (x - 1) + \left(-\frac{11}{12}\right) * (x - 1)(x - 4) =$$

$$= 2 + \frac{2}{3}x - \frac{2}{3} + \left(-\frac{11}{12}\right) * (x^2 - 5x + 4) =$$

$$= \frac{24}{12} + \frac{8}{12}x - \frac{8}{12} - \frac{11}{12}x^2 + \frac{55}{12}x - \frac{44}{12} =$$

$$= -\frac{11}{12}x^2 + \frac{21}{4}x - \frac{7}{3}$$



Příklad 7:

i	x_i	$y(x_i)$
0	2	1
1	3	2
2	4	-1
3	5	0

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(2-5)} = \frac{(x^2-3x-4x+12)(x-5)}{(-1)*(-2)*(-3)}$$
$$= \frac{(x^2-7x+12)(x-5)}{-6} = \frac{x^3-7x^2-5x^2+12x+35x-60}{-6}$$
$$= \frac{x^3-12x^2+47x-60}{-6}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x-2)(x-4)(x-5)}{(3-2)(3-4)(3-5)} = \frac{(x^2-2x-4x+8)(x-5)}{1*(-1)*(-2)}$$
$$= \frac{x^3-6x^2-5x^2+8x+30x-40}{2} = \frac{x^3-9x^2+23x-15}{2}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)} = \frac{(x^2-2x-3x+6)(x-5)}{2*1*(-1)}$$
$$= \frac{x^3-5x^2-5x^2+6x+25x-30}{-2} = \frac{x^3-10x^2+31x-30}{-2}$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x-2)(x-3)(x-4)}{(5-2)(5-3)(5-4)} = \frac{(x^2-2x-3x+6)(x-4)}{3*2*1}$$
$$= \frac{x^3-5x^2-4x^2+6x+20x-24}{6} = \frac{x^3-9x^2+26x-24}{6}$$

$$P(x) = (-1)*L_0(x) + 3L_1(x) + 0L_2(x) + 2L_3(x) =$$

$$= 1*\frac{x^3-12x^2+47x-60}{-6} + 2*\frac{x^3-9x^2+23x-15}{2} + (-1)*\frac{x^3-10x^2+31x-30}{-2} + 0 =$$

$$= \frac{-x^3+12x^2-47x+60}{6} + \frac{6x^3-66x^2+228x-240}{6} + \frac{3x^3-30x^2+93x-90}{6} =$$

$$= \frac{8x^3-84x^2+274x-270}{6} =$$

$$= \frac{4}{3}x^3 - 14x^2 + \frac{137}{3}x - 45$$

Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$	$y[x_i,x_{i+1},x_{i+2},x_{i+3}]$
2	1			
		$\frac{2-1}{3-2}=1$		
3	2		$\frac{-3-1}{4-2}=-2$	
		$\frac{-1-2}{4-3}=-3$		$\frac{2+2}{5-2}=\frac{4}{3}$
4	-1		$\frac{1+3}{5-3}=2$	
		$\frac{0+1}{5-4}=1$		
5	0			

$$P(x)=1+1*(x-2)+(-2)*(x-2)(x-3)+\frac{4}{3}(x-2)(x-3)(x-4)=$$

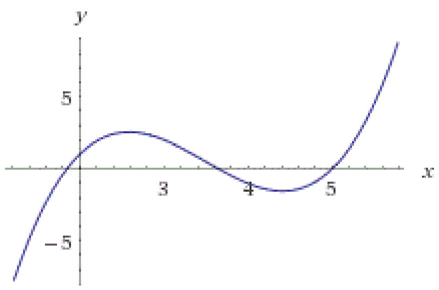
$$=1+x-2+(-2)*(x^2-5x+6)+\frac{4}{3}*(x^2-5x+6)(x-4)=$$

$$=1+x-2-2x^2+10x-12+\frac{4}{3}*(x^3-5x^2+6x-4x^2+20x-24)=$$

$$=-13+11x-2x^2+\frac{4}{3}*(x^3-9x^2+26x-24)=$$

$$=\frac{-39+33x-6x^2+4x^3-36x^2+104x-96}{3}=$$

$$=\frac{4}{3}x^3-14x^2+\frac{137}{3}x-45$$



Příklad 8:

i	x_i	$y(x_i)$
0	3	2
1	5	1
2	7	0

Langrangeův polynom:

$$L_0(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} = \frac{(x - 5)(x - 7)}{(3 - 5)(3 - 7)} = \frac{(x^2 - 7x - 5x + 35)}{(-2) * (-4)} = \frac{(x^2 - 12x + 35)}{8}$$

$$L_1(x) = \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} = \frac{(x - 3)(x - 7)}{(5 - 3)(5 - 7)} = \frac{(x^2 - 7x - 3x + 21)}{2 * (-2)} = \frac{(x^2 - 10x + 21)}{-4}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} = \frac{(x - 3)(x - 5)}{(7 - 3)(7 - 5)} = \frac{(x^2 - 5x - 3x + 15)}{4 * 2} = \frac{(x^2 - 8x + 15)}{8}$$

$$P(x) = 2 * L_0(x) + 1 * L_1(x) + 0 * L_2(x) =$$

$$= 2 * \frac{(x^2 - 12x + 35)}{8} + 1 * \frac{(x^2 - 10x + 21)}{-4} + 0 * \frac{(x^2 - 8x + 15)}{8} =$$

$$= \frac{x^2 - 12x + 35 - x^2 + 10x - 21}{4} =$$

$$= -\frac{1}{2}x + \frac{7}{2}$$

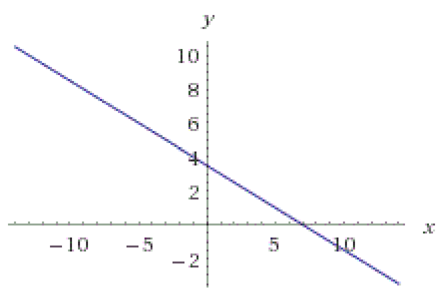
Newtonův polynom:

x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
3	2		
		$\frac{1-2}{5-3} = -\frac{1}{2}$	
5	1		$\frac{-\frac{1}{2}+\frac{1}{2}}{7-3} = 0$
		$\frac{0-1}{7-5} = -\frac{1}{2}$	
7	0		

$$P(x) = y_0 + y[x_0,x_1] * (x - x_0) + y[x_0,x_1,x_2] * (x - x_0)(x - x_1) =$$

$$= 2 + \left(-\frac{1}{2}\right) * (x-3) + 0 * (x-3)(x-5) =$$

$$= -\frac{1}{2}x + \frac{7}{2}$$



Příklad 9:

i	x_i	$y(x_i)$
0	0	3
1	1	0
2	5	2

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-1)(x-5)}{(0-1)(0-5)} = \frac{x^2-6x+5}{5}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-5)}{(1-0)(1-5)} = \frac{x^2-5x}{-4}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)(x-1)}{(5-0)(5-1)} = \frac{(x^2-x)}{20}$$

$$P(x) = 3 * L_0(x) + 0 * L_1(x) + 2 * L_2(x) =$$

$$= 3 * \frac{x^2-6x+5}{5} + 0 * \frac{x^2-5x}{-4} + 2 * \frac{x^2-x}{20} =$$

$$= \frac{6x^2-36x+30+x^2-x}{10} =$$

$$= \frac{7}{10}x^2 - \frac{37}{10}x + 3$$

Newtonův polynom:

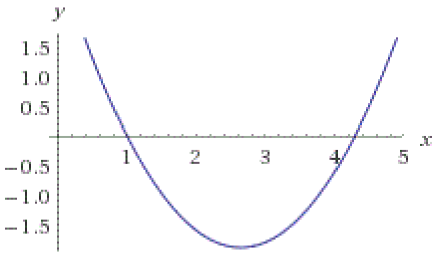
x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
0	3		
		$\frac{0-3}{1-0}=-3$	
1	0		$\frac{\frac{1}{2}+3}{5-0}=\frac{7}{10}$
		$\frac{2-0}{5-1}=\frac{1}{2}$	
5	2		

$$P(x) = y_0 + y[x_0,x_1] * (x - x_0) + y[x_0,x_1,x_2] * (x - x_0)(x - x_1) =$$

$$= 3 + (-3) * (x - 0) + \frac{7}{10} * (x - 0)(x - 1) =$$

$$= 3 - 3x + \frac{7}{10} * (x^2 - x) =$$

$$= \frac{7}{10}x^2 - \frac{37}{10}x + 3$$



Příklad 10:

i	x_i	$y(x_i)$
0	1	2
1	2	1
2	3	0

Langrangeův polynom:

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{x^2-5x+6}{2}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{x^2-4x+3}{-1}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{x^2-3x+2}{2}$$

$$P(x) = 2 * L_0(x) + 1 * L_1(x) + 0 * L_2(x) =$$

$$= 2 * \frac{x^2-5x+6}{2} + 1 * \frac{x^2-4x+3}{-1} + 0 * \frac{x^2-3x+2}{2} =$$

$$= x^2 - 5x + 6 - x^2 + 4x - 3 =$$

$$= -x + 3$$

Newtonův polynom:

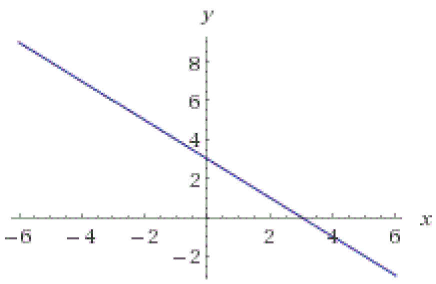
x_i	y_i	$y[x_i,x_{i+1}]$	$y[x_i,x_{i+1},x_{i+2}]$
1	2 ↘		
		$\frac{1-2}{2-1}=-1$ ↘	
2	1 ↗↘		$\frac{-1+1}{3-1}=0$
		$\frac{0-1}{3-2}=-1$ ↗	
3	0 ↗		

$$P(x) = y_0 + y[x_0,x_1]*(x-x_0) + y[x_0,x_1,x_2]*(x-x_0)(x-x_1) =$$

$$= 0 + (-1)*(x-1) + 0*(x-1)(x-0) =$$

$$= 2 - x + 1 =$$

$$= -x + 3$$



Seznam použité literatury

[1] Dalík, J.: Numerické metody, Akademické nakladatelství CERM s.r.o. Brno, Brno 1997

Za spolupráce a pod vedením Mgr. Ireny Hinterleitner, které tímto děkuji.

V Brně 2016