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LU-rozklad



Teoretický úvod

LU rozklad nám říká, že čtvercovou regulární matici lze rozložit na součin matic $A=L*U$, kde L je dolní trojúhelníková matice s jedničkami na hlavní diagonále a U je horní trojúhelníková matice a slouží k řešení soustavy lineárních rovnic. LU rozklad lze použít za předpokladu, že původní matice A je čtvercová. Toto si ukážeme v následujících příkladech.

Příklad 1:

Mějme rovnice:

$$\begin{aligned} x_1 - 2x_2 &= 1 \\ 2x_1 - x_2 + x_3 &= 2 \\ -x_1 + 2x_2 - x_3 &= 0 \end{aligned}$$

Vytvoříme matici A.

$$A = \left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 2 \\ -1 & 2 & -1 & 0 \end{array} \right)$$

Upravíme matici do tzv. schodového tvaru, kdy se pod hlavní diagonálou vyskytují pouze nuly.

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 2 \\ -1 & 2 & -1 & 0 \end{array} \right) \begin{array}{l} |* (-2) \\ \leftarrow + \\ (m_{21} = 2) \end{array} \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & 0 \\ -1 & 2 & -1 & 0 \end{array} \right) \begin{array}{l} |* 1 \\ (m_{31} = -1) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{array} \right) \begin{array}{l} (m_{32} = 0) \end{array}$$

Vyšetření kořenů rovnic:

$$\begin{aligned} -x_3 &= 1 \Rightarrow x_3 = -1 \\ 3x_2 + 1x_3 &= 0 \Rightarrow x_2 = \frac{1}{3} \\ x_1 + (-2) * x_2 + 0 * x_3 &= 1 \Rightarrow x_1 = \frac{5}{3} \end{aligned}$$

Získali jsme tedy matici $U = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & -1 \end{pmatrix}$ a matici $L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

Jako zkoušku tyto matice vynásobíme, abychom potvrdili správnost rozložení do tvaru $A=L*U$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & -2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 * 1 + 0 * 0 + 0 * 0 & 1 * (-2) + 0 * 3 + 0 * 0 & 1 * 0 + 0 * 1 + 0 * (-1) \\ 2 * 1 + 1 * 0 + 0 * 0 & 2 * (-2) + 1 * 3 + 0 * 0 & 2 * 0 + 1 * 1 + 0 * (-1) \\ (-1) * 1 + 0 * 0 + 1 * 0 & (-1) * (-2) + 0 * 3 + 1 * 0 & (-1) * 0 + 0 * 1 + 1 * (-1) \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & -2 & 0 \\ 2 & -1 & 1 \\ -1 & 2 & -1 \end{pmatrix} \Rightarrow A=L*U$$

Příklad 2:

$$A = \left(\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 2 & -1 & 1 & 2 \\ -2 & 1 & 0 & 1 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 2 & -1 & 1 & 2 \\ -2 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} |* (-2) \\ \swarrow + \\ (m_{21} = 2) \end{array} &\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -7 & 3 & -4 \\ -2 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} |* 2 \\ (m_{31} = -2) \\ \swarrow + \end{array} &\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -7 & 3 & -4 \\ 0 & 7 & -2 & 7 \end{array} \right) \begin{array}{l} (m_{32} = -1) \\ |* 1 \\ \swarrow + \end{array} &\sim \\ &\sim \left(\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & -7 & 3 & -4 \\ 0 & 0 & 1 & 3 \end{array} \right) \end{aligned}$$

$$x_3 = 3$$

$$-7x_2 + 3x_3 = -4 \Rightarrow x_2 = \frac{13}{7}$$

$$x_1 + 3x_2 + (-1) * x_3 = 3 \Rightarrow x_1 = \frac{3}{7}$$

$$U = \begin{pmatrix} 1 & 3 & -1 \\ 0 & -7 & 3 \\ 0 & 0 & 1 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 1 & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -2 & 1 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 3 & -1 \\ 0 & -7 & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & -1 & 1 \\ -1 & 2 & -1 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 3:

$$A = \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & -1 & 2 & 3 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 2 & -1 & 1 & 0 \\ 1 & -1 & 2 & 3 \end{array} \right) \begin{array}{l} | * (-2) \\ \leftarrow + \\ (m_{21} = 2) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -7 & 5 & -2 \\ 1 & -1 & 2 & 3 \end{array} \right) \begin{array}{l} | * (-1) \\ (m_{31} = 1) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -7 & 5 & -2 \\ 0 & -4 & 0 & 2 \end{array} \right) \begin{array}{l} (m_{32} = \frac{4}{7}) \\ | * (-\frac{4}{7}) \\ \leftarrow + \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 3 & -2 & 1 \\ 0 & -7 & 5 & -2 \\ 0 & 0 & -\frac{20}{7} & \frac{22}{7} \end{array} \right)$$

$$-\frac{20}{7}x_3 = \frac{22}{7} \Rightarrow x_3 = -\frac{11}{10}$$

$$-7x_2 + 5x_3 = -2 \Rightarrow x_2 = -\frac{1}{2}$$

$$x_1 + 3x_2 + (-2) * x_3 = 1 \Rightarrow x_1 = -\frac{27}{2}$$

$$U = \left(\begin{array}{ccc} 1 & 3 & -2 \\ 0 & -7 & 5 \\ 0 & 0 & -\frac{20}{7} \end{array} \right), L = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{4}{7} & 1 \end{array} \right)$$

Zkouška:

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{4}{7} & 1 \end{array} \right) * \left(\begin{array}{ccc} 1 & 3 & -2 \\ 0 & -7 & 5 \\ 0 & 0 & -\frac{20}{7} \end{array} \right) = \left(\begin{array}{ccc} 1 & 3 & -2 \\ 2 & -1 & 1 \\ 1 & -1 & 2 \end{array} \right) \Rightarrow A = L * U$$

Příklad 4:

$$A = \left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 5 & 1 & 3 & 1 \\ -2 & 0 & 1 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 5 & 1 & 3 & 1 \\ -2 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} | * (-5) \\ \leftarrow + \\ (m_{21} = 5) \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 6 & 17 & -4 \\ -2 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} | * 2 \\ (m_{31} = -2) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 6 & 17 & -4 \\ 0 & -2 & 9 & 3 \end{array} \right) \begin{array}{l} (m_{32} = -\frac{1}{3}) \\ | * \frac{1}{3} \\ \leftarrow + \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -1 & 4 & 1 \\ 0 & 6 & 17 & -4 \\ 0 & 0 & \frac{44}{3} & \frac{5}{3} \end{array} \right)$$

$$\frac{44}{3}x_3 = \frac{5}{3} \Rightarrow x_3 = \frac{5}{44}$$

$$6x_2 + 17x_3 = -4 \Rightarrow x_2 = \frac{87}{88}$$

$$x_1 + (-1) * x_2 + 4x_3 = 3 \Rightarrow x_1 = \frac{311}{88}$$

$$U = \begin{pmatrix} 1 & -1 & 4 \\ 0 & 6 & 17 \\ 0 & 0 & \frac{44}{3} \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & \frac{1}{3} & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & \frac{1}{3} & 1 \end{pmatrix} * \begin{pmatrix} 1 & -1 & 4 \\ 0 & 6 & 17 \\ 0 & 0 & \frac{44}{3} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 4 \\ 5 & 1 & 3 \\ -2 & 0 & 1 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 5:

$$A = \left(\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 3 & 2 & -1 & 0 \\ 1 & 1 & 2 & 1 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 3 & 2 & -1 & 0 \\ 1 & 1 & 2 & 1 \end{array} \right) & \begin{array}{l} \left| * \left(-\frac{3}{2} \right) \right. \\ \leftarrow + \\ (m_{21} = \frac{3}{2}) \end{array} \sim \left(\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 0 & \frac{1}{2} & 2 & -6 \\ 1 & 1 & 2 & 1 \end{array} \right) \begin{array}{l} \left| * \left(-\frac{1}{2} \right) \right. \\ (m_{31} = \frac{1}{2}) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 0 & \frac{1}{2} & 2 & -6 \\ 0 & \frac{1}{2} & 3 & -1 \end{array} \right) \begin{array}{l} (m_{32} = -1) \\ \left| * 1 \right. \\ \leftarrow + \end{array} \sim \\ & \sim \left(\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 0 & \frac{1}{2} & 2 & -6 \\ 0 & 0 & 1 & 5 \end{array} \right) \end{aligned}$$

$$x_3 = 5$$

$$\frac{1}{2}x_2 + 2x_3 = -6 \Rightarrow x_2 = -32$$

$$2x_1 + x_2 + (-2) * x_3 = 4 \Rightarrow x_1 = 23$$

$$U = \begin{pmatrix} 2 & 1 & -2 \\ 0 & \frac{1}{2} & 2 \\ 0 & 0 & 1 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & -1 & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & -1 & 1 \end{pmatrix} * \begin{pmatrix} 2 & 1 & -2 \\ 0 & \frac{1}{2} & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & -2 \\ 3 & 2 & -1 \\ 1 & 1 & 2 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 6:

$$A = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 1 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 1 \end{array} \right) \begin{array}{l} |* (-4) \\ \leftarrow + \\ (m_{21} = 4) \end{array} &\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 2 \\ 7 & 8 & 9 & 1 \end{array} \right) \begin{array}{l} |* (-7) \\ (m_{31} = 7) \\ \leftarrow + \end{array} &\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 2 \\ 0 & -6 & -12 & 1 \end{array} \right) \begin{array}{l} (m_{32} = 2) \\ |* (-2) \\ \leftarrow + \end{array} \\ &\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 2 \\ 0 & 0 & 0 & -3 \end{array} \right) \end{aligned}$$

Tato soustava rovnic nemá řešení, avšak L-U rozklad je i tak možný provést.

$$U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 7:

$$A = \left(\begin{array}{ccc|c} 3 & 5 & 4 & 3 \\ 6 & 2 & 8 & 0 \\ 1 & 7 & 9 & 4 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 3 & 5 & 4 & 3 \\ 6 & 2 & 8 & 0 \\ 1 & 7 & 9 & 4 \end{array} \right) \begin{array}{l} | * (-2) \\ \leftarrow + \\ (m_{21} = 2) \end{array} &\sim \left(\begin{array}{ccc|c} 3 & 5 & 4 & 3 \\ 0 & -8 & 0 & -6 \\ 1 & 7 & 9 & 4 \end{array} \right) \begin{array}{l} | * \left(-\frac{1}{3}\right) \\ (m_{31} = \frac{1}{3}) \\ \leftarrow + \end{array} &\sim \left(\begin{array}{ccc|c} 3 & 5 & 4 & 3 \\ 0 & -8 & 0 & -6 \\ 0 & \frac{16}{3} & \frac{23}{3} & 6 \end{array} \right) \begin{array}{l} (m_{32} = \frac{2}{3}) \\ | * \left(-\frac{2}{3}\right) \\ \leftarrow + \end{array} &\sim \\ &\sim \left(\begin{array}{ccc|c} 3 & 5 & 4 & 3 \\ 0 & -8 & 0 & -6 \\ 0 & 0 & \frac{23}{3} & 2 \end{array} \right) \end{aligned}$$

$$\frac{23}{3}x_3 = 2 \Rightarrow x_3 = \frac{6}{23}$$

$$-8x_2 + 0x_3 = -6 \Rightarrow x_2 = \frac{3}{4}$$

$$3x_1 + 5x_2 + 4x_3 = 3 \Rightarrow x_1 = \frac{-165}{92}$$

$$U = \begin{pmatrix} 3 & 5 & 4 \\ 0 & -8 & 0 \\ 0 & 0 & \frac{23}{3} \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{1}{3} & \frac{2}{3} & 1 \end{pmatrix} * \begin{pmatrix} 3 & 5 & 4 \\ 0 & -8 & 0 \\ 0 & 0 & \frac{23}{3} \end{pmatrix} = \begin{pmatrix} 3 & 5 & 4 \\ 6 & 2 & 8 \\ 1 & 7 & 9 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 8:

$$A = \left(\begin{array}{ccc|c} 2 & 4 & 6 & -1 \\ -2 & 3 & 1 & 2 \\ 5 & 6 & 2 & 0 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 2 & 4 & 6 & -1 \\ -2 & 3 & 1 & 2 \\ 5 & 6 & 2 & 0 \end{array} \right) & \begin{array}{l} | * 1 \\ \swarrow + \end{array} \sim \left(\begin{array}{ccc|c} 2 & 4 & 6 & -1 \\ 0 & 7 & 7 & 1 \\ 5 & 6 & 2 & 0 \end{array} \right) \begin{array}{l} | * \left(-\frac{5}{2}\right) \\ (m_{31} = \frac{5}{2}) \\ \swarrow + \end{array} \sim \left(\begin{array}{ccc|c} 2 & 4 & 6 & -1 \\ 0 & 7 & 7 & 1 \\ 0 & -4 & -13 & -\frac{5}{2} \end{array} \right) \begin{array}{l} (m_{32} = -\frac{4}{7}) \\ | * \frac{4}{7} \\ \swarrow + \end{array} \sim \\ & \sim \left(\begin{array}{ccc|c} 2 & 4 & 6 & -1 \\ 0 & 7 & 7 & 1 \\ 0 & 0 & -9 & -\frac{27}{14} \end{array} \right) \end{aligned}$$

$$-9x_3 = -\frac{27}{14} \Rightarrow x_3 = \frac{3}{14}$$

$$7x_2 + x_3 = 1 \Rightarrow x_2 = \frac{11}{98}$$

$$2x_1 + 4x_2 + 6x_3 = -1 \Rightarrow x_1 = \frac{67}{49}$$

$$U = \begin{pmatrix} 2 & 4 & 6 \\ 0 & 7 & 7 \\ 0 & 0 & -9 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{5}{2} & -\frac{4}{7} & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ \frac{5}{2} & -\frac{4}{7} & 1 \end{pmatrix} * \begin{pmatrix} 2 & 4 & 6 \\ 0 & 7 & 7 \\ 0 & 0 & -9 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 6 \\ -2 & 3 & 1 \\ 5 & 6 & 2 \end{pmatrix} \Rightarrow A = L * U$$

Příklad 9:

$$A = \left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 4 & 5 & 1 & 2 \\ 6 & 2 & 3 & 1 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 4 & 5 & 1 & 2 \\ 6 & 2 & 3 & 1 \end{array} \right) \begin{array}{l} |* (-4) \\ \leftarrow + \\ (m_{21} = 4) \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -7 & -7 & -2 \\ 6 & 2 & 3 & 1 \end{array} \right) \begin{array}{l} |* (-6) \\ (m_{31} = 6) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -7 & -7 & -2 \\ 0 & -16 & -9 & -5 \end{array} \right) \begin{array}{l} (m_{32} = \frac{16}{7}) \\ |* (-\frac{16}{7}) \\ \leftarrow + \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & -7 & -7 & -2 \\ 0 & 0 & 7 & -\frac{3}{7} \end{array} \right)$$

$$7x_3 = -\frac{3}{7} \Rightarrow x_3 = \frac{3}{49}$$

$$-7x_2 + (-7) * x_3 = -2 \Rightarrow x_2 = \frac{11}{49}$$

$$x_1 + 3x_2 + 2x_3 = 1 \Rightarrow x_1 = \frac{10}{49}$$

$$U = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -7 & -7 \\ 0 & 0 & 7 \end{pmatrix}, L = \begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 6 & \frac{16}{7} & 1 \end{pmatrix}$$

Zkouška:

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 6 & \frac{16}{7} & 1 \end{pmatrix} * \begin{pmatrix} 1 & 3 & 2 \\ 0 & -7 & -7 \\ 0 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 5 & 1 \\ 6 & 2 & 3 \end{pmatrix} \Rightarrow A=L*U$$

Příklad 10:

$$A = \left(\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 3 & 5 & 6 & 1 \\ 1 & 4 & 3 & 2 \end{array} \right)$$

$$\begin{aligned} \left(\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 3 & 5 & 6 & 1 \\ 1 & 4 & 3 & 2 \end{array} \right) & \begin{array}{l} | * \left(-\frac{3}{2} \right) \\ \leftarrow + \\ (m_{21} = \frac{3}{2}) \end{array} \sim \left(\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 0 & -1 & \frac{9}{2} & 1 \\ 1 & 4 & 3 & 2 \end{array} \right) \begin{array}{l} | * \left(-\frac{1}{2} \right) \\ (m_{31} = \frac{1}{2}) \\ \leftarrow + \end{array} \sim \left(\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 0 & -1 & \frac{9}{2} & 1 \\ 0 & 2 & \frac{5}{2} & \frac{3}{2} \end{array} \right) \begin{array}{l} (m_{32} = 2) \\ | * (-2) \\ \leftarrow + \end{array} \sim \\ & \sim \left(\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 0 & -1 & \frac{9}{2} & 1 \\ 0 & 0 & \frac{23}{2} & -\frac{1}{2} \end{array} \right) \end{aligned}$$

$$\frac{23}{2}x_3 = -\frac{1}{2} \Rightarrow x_3 = -\frac{1}{23}$$

$$-x_2 + \frac{9}{2}x_3 = 1 \Rightarrow x_2 = -\frac{55}{46}$$

$$2x_1 + 4x_2 + 1x_3 = 0 \Rightarrow x_1 = \frac{111}{46}$$

$$U = \left(\begin{array}{ccc} 2 & 4 & 1 \\ 0 & -1 & \frac{9}{2} \\ 0 & 0 & \frac{23}{2} \end{array} \right), L = \left(\begin{array}{ccc} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 2 & 1 \end{array} \right)$$

Zkouška:

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 2 & 1 \end{array} \right) * \left(\begin{array}{ccc} 2 & 4 & 1 \\ 0 & -1 & \frac{9}{2} \\ 0 & 0 & \frac{23}{2} \end{array} \right) = \left(\begin{array}{ccc} 2 & 4 & 1 \\ 3 & 5 & 6 \\ 1 & 4 & 3 \end{array} \right) \Rightarrow A = L * U$$

Seznam použité literatury

[1] Dalík, J.: Numerické metody, Akademické nakladatelství CERM s.r.o. Brno, Brno 1997

Za spolupráce a pod vedením Mgr. Ireny Hinterleitner, které tímto děkuji.

V Brně 2015