Řešení přeurčitých soustav lineárních algebraických rovnic metodou nejmenších čtverců

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Teoretický úvod

Abychom na matici A mohli provést LU-rozklad je nutné, aby matice A byla čtvercová a regulární. LU-rozklad je způsob jakým můžeme matici A zapsat jako součin dvou matic L a U. Matice L je dolní trojúhelníková matice s jedničkami na hlavní diagonále a matice U je horní trojúhelníková matice.

LU-rozklad se používá k řešení systému lineárních rovnic $Ax = b$ stejně jako Gausova eliminační metoda, ovšem LU-rozklad je mnohem efektivnější v situaci, že máme sérii výpočtů, ve kterých máme více vektorů pravých stran $b^{(i)}, kde\ i = 1, 2, ...$ Pro každé konkrétní $b^{(i)}$ dopočítáme řešení $x^{(i)}$.

Pokud k řešení soustavy $Ax = b$ využijeme rozkladu matice A na součin matic L a U, můžeme celý postup výpočtu zapsat jako $Ax = (LU)x = L(Ux) = Ly = b$.

Řešené příklady

Příklad 1.

Mějme soustavu rovnic:

$$\begin{aligned} y &= 2 \\ x &= 1 \\ x + y &= 4 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} y = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 2 \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = 1 \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = 5$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = 1 \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 2 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = 6$$

$$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 1 & 2 & 6 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \qquad |D_1| = \begin{vmatrix} 5 & 1 \\ 6 & 2 \end{vmatrix} = 4 \qquad |D_2| = \begin{vmatrix} 2 & 5 \\ 1 & 6 \end{vmatrix} = 7$$

$$x = \frac{|D_1|}{|D|} = \frac{4}{3}$$

$$y = \frac{|D_2|}{|D|} = \frac{7}{3}$$

Dosazení:

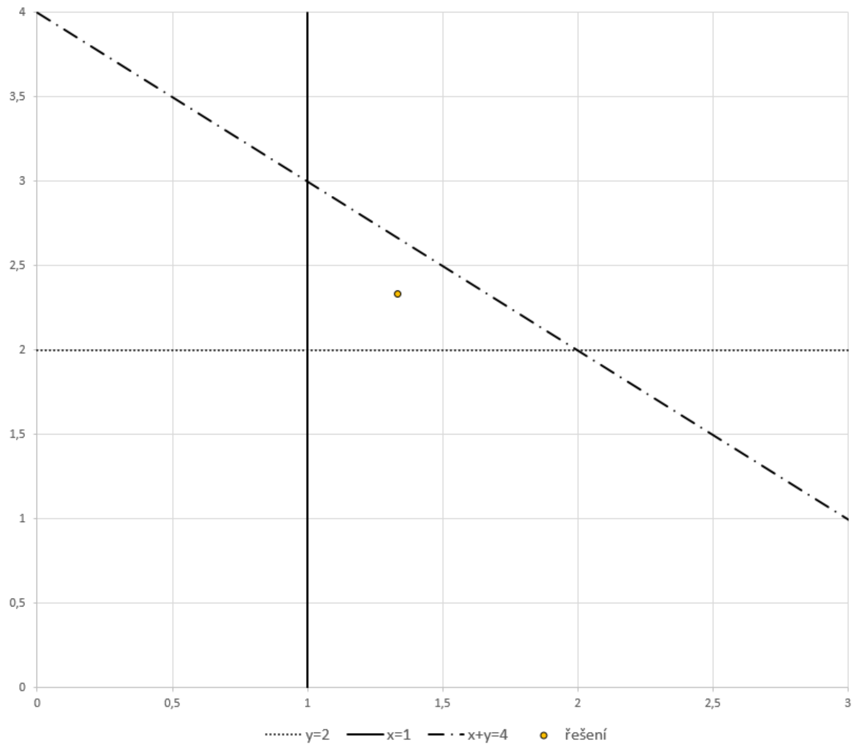
$$\frac{7}{3} = \frac{7}{3}$$
$$\frac{4}{3} = \frac{4}{3}$$
$$\frac{4}{3} + \frac{7}{3} = \frac{11}{3}$$

$$\varphi = (2; 1; 4)$$
$$\varphi^* = \left(\frac{7}{3}; \frac{4}{3}; \frac{11}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(2 - \frac{7}{3}\right)^2 + \left(1 - \frac{4}{3}\right)^2 + \left(4 - \frac{11}{3}\right)^2} = 0,5774$$

Graf



Příklad 2.

Mějme soustavu rovnic:

$$\begin{aligned} x &= 0 \\ y &= 1 \\ x + y &= 3 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \ \varphi^{(2)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \ \varphi = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 2 \ \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = 1 \ \left\langle \varphi^{(1)}; \varphi \right\rangle = 3$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = 1 \ \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 2 \ \left\langle \varphi^{(2)}; \varphi \right\rangle = 4$$

$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 1 & 2 & 4 \end{array}\right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3 \qquad |D_1| = \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = 2 \qquad |D_2| = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = 5$$

$$x = \frac{|D_1|}{|D|} = \frac{2}{3}$$

$$y = \frac{|D_2|}{|D|} = \frac{5}{3}$$

Dosazení:

$$\frac{2}{3} = \frac{2}{3}$$

$$\frac{5}{3} = \frac{5}{3}$$

$$\frac{2}{3} + \frac{5}{3} = \frac{7}{3}$$

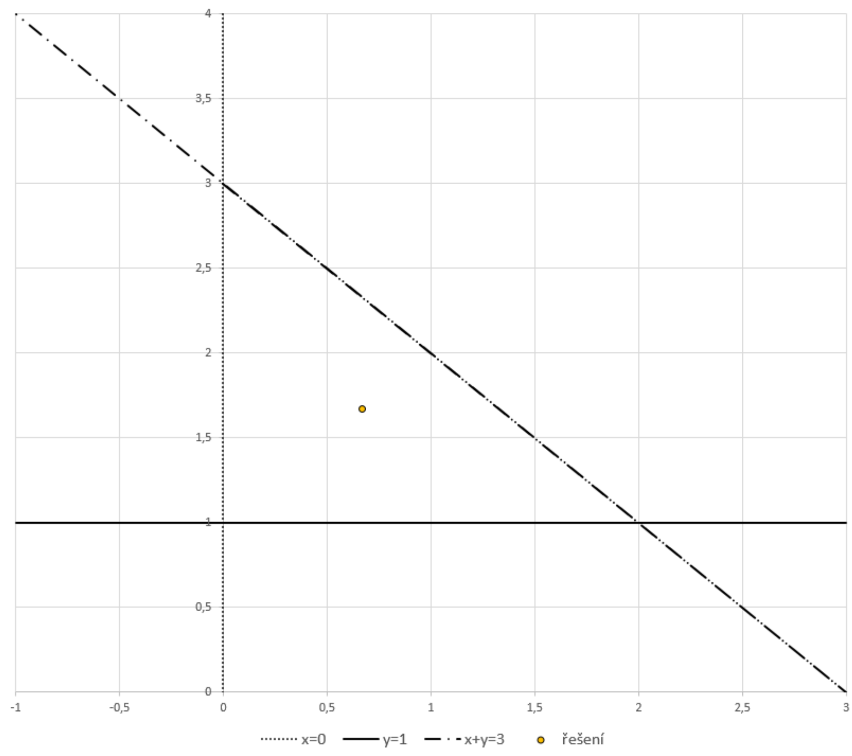
$$\varphi = (0;1;3)$$

$$\varphi^* = \left(\frac{2}{3};\frac{5}{3};\frac{7}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(0-\frac{2}{3}\right)^2 + \left(1-\frac{5}{3}\right)^2 + \left(3-\frac{7}{3}\right)^2} = 1,1547$$

Graf



Příklad 3.

Mějme soustavu rovnic:

$$\begin{aligned} x &= 0 \\ y &= 3 \\ -x+y &= 0 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \; \varphi^{(2)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \; \varphi = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 2 \;\; \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = -1 \;\; \left\langle \varphi^{(1)}; \varphi \right\rangle = 0$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = -1 \;\; \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 2 \;\; \left\langle \varphi^{(2)}; \varphi \right\rangle = 3$$

$$\left[\begin{array}{cc|c} 2 & -1 & 0 \\ -1 & 2 & 3 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 \qquad |D_1| = \begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} = 3 \qquad |D_2| = \begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} = 6$$

$$x = \frac{|D_1|}{|D|} = \frac{3}{3} = 1$$

$$y = \frac{|D_2|}{|D|} = \frac{6}{3} = 2$$

Dosazení:

$$1 = 1$$

$$2 = 2$$

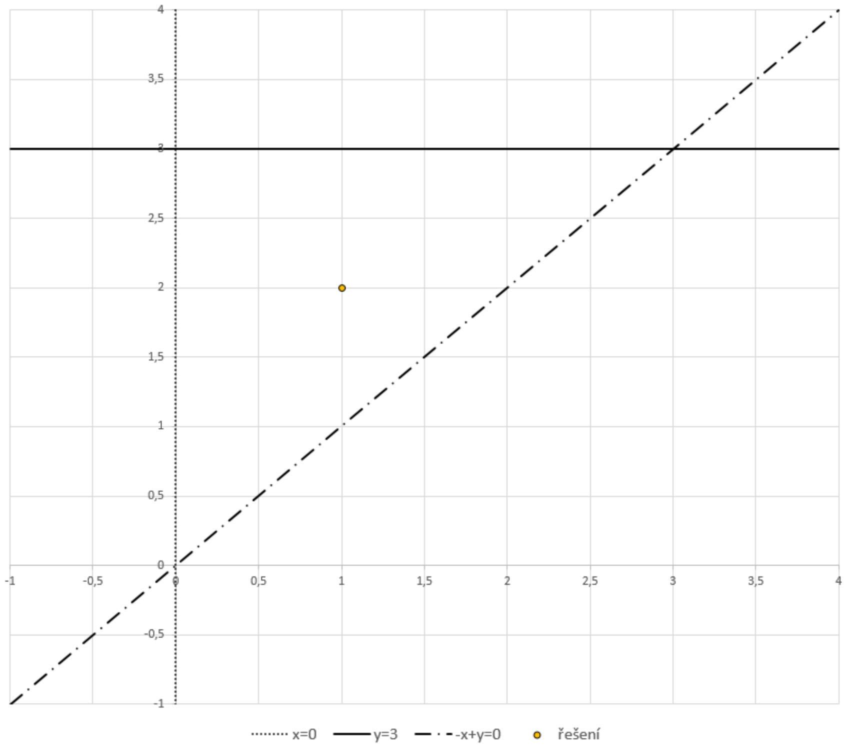
$$-1 + 2 = 1$$

$$\varphi = (0;3;0)$$
$$\varphi^* = (1;2;1)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{(0-1)^2 + (3-2)^2 + (0-1)^2} = 1,7321$$

Graf



Příklad 4.

Mějme soustavu rovnic:

$$\begin{aligned} x &= 4 \\ y &= 1 \\ -x + y &= 0 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \; \varphi^{(2)} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \; \varphi = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 2 \;\; \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = -1 \;\; \left\langle \varphi^{(1)}; \varphi \right\rangle = 4$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = -1 \;\; \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 2 \;\; \left\langle \varphi^{(2)}; \varphi \right\rangle = 1$$

$$\left[\begin{array}{cc|c} 2 & -1 & 4 \\ -1 & 2 & 1 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3 \qquad |D_1| = \begin{vmatrix} 4 & -1 \\ 1 & 2 \end{vmatrix} = 9 \qquad |D_2| = \begin{vmatrix} 2 & 4 \\ -1 & 1 \end{vmatrix} = 6$$

$$x = \frac{|D_1|}{|D|} = \frac{9}{3} = 3$$

$$y = \frac{|D_2|}{|D|} = \frac{6}{3} = 2$$

Dosazení:

$$3 = 3$$

$$2 = 2$$

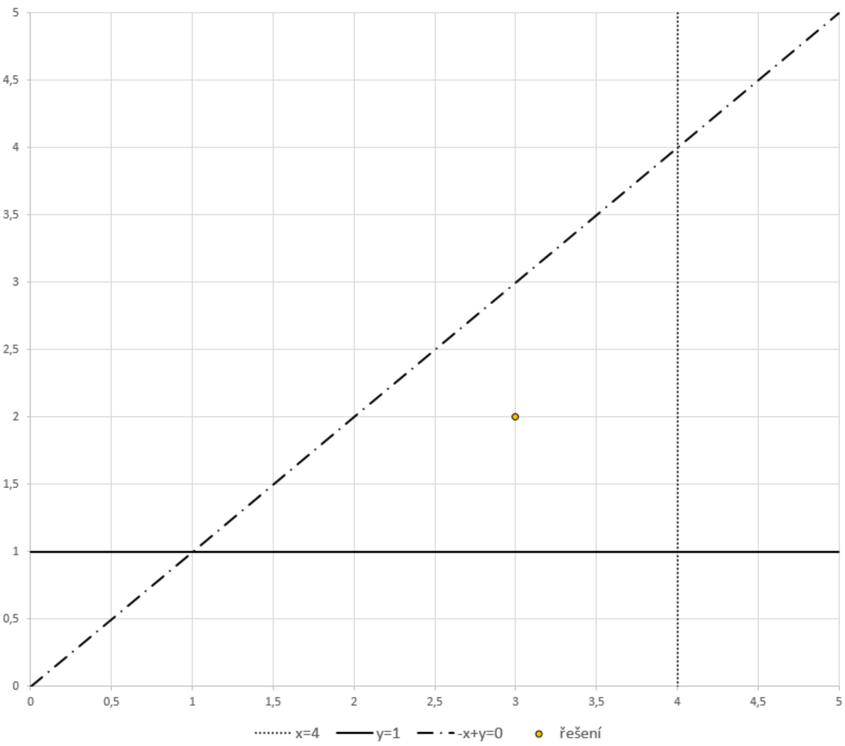
$$-3 + 2 = -1$$

$$\varphi = (4; 1; 0)$$
$$\varphi^* = (3; 2; -1)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{(4-3)^2 + (1-2)^2 + (0+1)^2} = 1,7321$$

Graf



Příklad 5.

Mějme soustavu rovnic:

$$\begin{aligned} 2x + y &= 4 \\ x + y &= 4 \\ y &= 0 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 5 \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = 12$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = 8$$

$$\left[\begin{array}{cc|c} 5 & 3 & 12 \\ 3 & 3 & 8 \end{array}\right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 5 & 3 \\ 3 & 3 \end{vmatrix} = 6 \qquad |D_1| = \begin{vmatrix} 12 & 3 \\ 8 & 3 \end{vmatrix} = 12 \qquad |D_2| = \begin{vmatrix} 5 & 12 \\ 3 & 8 \end{vmatrix} = 4$$

$$x = \frac{|D_1|}{|D|} = \frac{12}{6} = 2$$

$$y = \frac{|D_2|}{|D|} = \frac{4}{6} = \frac{2}{3}$$

Dosazení:

$$2 \cdot 2 + \frac{2}{3} = \frac{14}{3}$$

$$2 + \frac{2}{3} = \frac{8}{3}$$

$$\frac{2}{3} = \frac{2}{3}$$

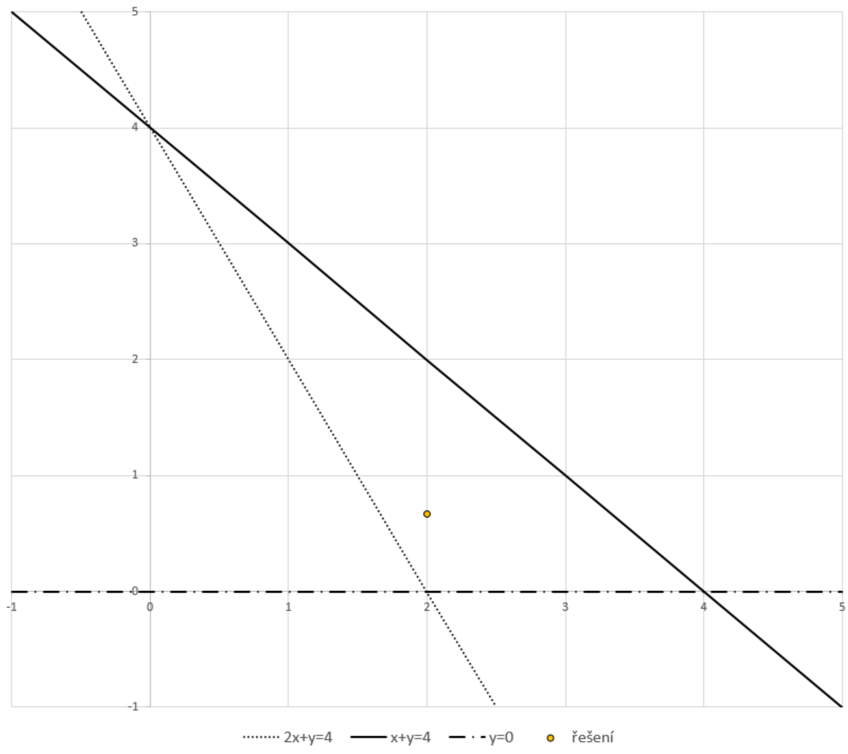
$$\varphi = (4; 4; 0)$$

$$\varphi^* = \left(\frac{14}{3}; \frac{8}{3}; \frac{2}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(4 - \frac{14}{3}\right)^2 + \left(4 - \frac{8}{3}\right)^2 + \left(0 - \frac{2}{3}\right)^2} = 1,6330$$

Graf



Příklad 6.

Mějme soustavu rovnic:

$$\begin{aligned} -x + y &= 0 \\ x + y &= 4 \\ y &= 0 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 2 \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = 0 \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = 4$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = 0 \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = 4$$

$$\left[\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & 3 & 4 \end{array}\right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 \qquad |D_1| = \begin{vmatrix} 4 & 0 \\ 4 & 3 \end{vmatrix} = 12 \qquad |D_2| = \begin{vmatrix} 2 & 4 \\ 0 & 4 \end{vmatrix} = 8$$

$$x = \frac{|D_1|}{|D|} = \frac{12}{6} = 2$$

$$y = \frac{|D_2|}{|D|} = \frac{8}{6} = \frac{4}{3}$$

Dosazení:

$$-2 + \frac{4}{3} = -\frac{2}{3}$$

$$2 + \frac{4}{3} = \frac{10}{3}$$

$$\frac{4}{3} = \frac{4}{3}$$

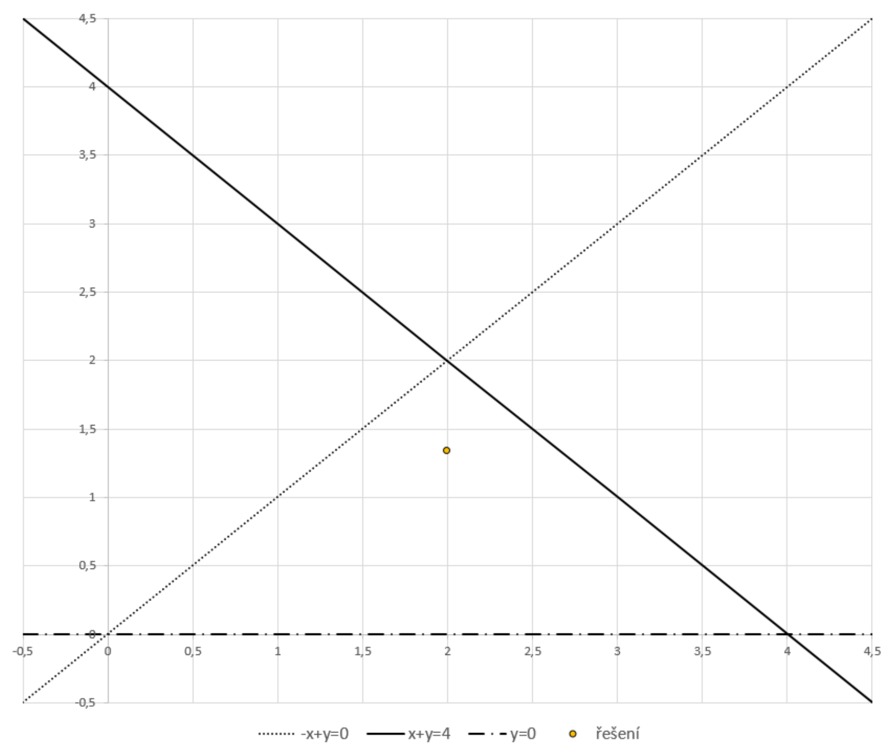
$$\varphi = (0; 4; 0)$$

$$\varphi^* = \left(-\frac{2}{3}; \frac{10}{3}; \frac{4}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(0 + \frac{2}{3}\right)^2 + \left(4 - \frac{10}{3}\right)^2 + \left(0 - \frac{4}{3}\right)^2} = 1,6330$$

Graf



Příklad 7.

Mějme soustavu rovnic:

$$\begin{aligned} x + y &= 1 \\ -x + y &= -3 \\ x &= -2 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} y = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \varphi = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 3 \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = 0 \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = 2$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = 0 \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 2 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = -2$$

$$\left[\begin{array}{cc|c} 3 & 0 & 2 \\ 0 & 2 & -2 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = 6 \qquad |D_1| = \begin{vmatrix} 2 & 0 \\ -2 & 2 \end{vmatrix} = 4 \qquad |D_2| = \begin{vmatrix} 3 & 2 \\ 0 & -2 \end{vmatrix} = -6$$

$$x = \frac{|D_1|}{|D|} = \frac{4}{6} = \frac{2}{3}$$

$$y = \frac{|D_2|}{|D|} = \frac{-6}{6} = -1$$

Dosazení:

$$\frac{2}{3} + (-1) = -\frac{1}{3}$$

$$-\frac{2}{3} + (-1) = -\frac{5}{3}$$

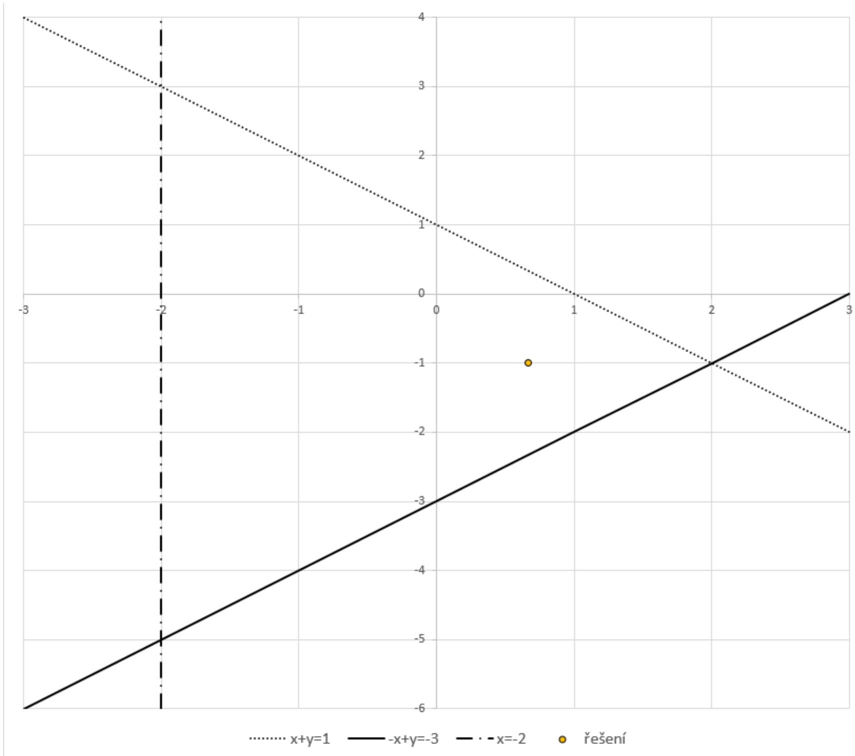
$$\frac{2}{3} = \frac{2}{3}$$

$$\varphi = (1; -3; -2)$$
$$\varphi^* = \left(-\frac{1}{3}; -\frac{5}{3}; \frac{2}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(1 + \frac{1}{3}\right)^2 + \left(-3 + \frac{5}{3}\right)^2 + \left(-2 - \frac{2}{3}\right)^2} = 3,2660$$

Graf



Příklad 8.

Mějme soustavu rovnic:

$$\begin{aligned} x + y &= 6 \\ -\frac{2}{3}x + y &= 0 \\ -4x + y &= 6 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} 1 \\ -\frac{2}{3} \\ -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} 1 \\ -\frac{2}{3} \\ -4 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = \frac{157}{9} \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = -\frac{11}{3} \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = -18$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = -\frac{11}{3} \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = 12$$

$$\left[\begin{array}{cc|c} \frac{157}{9} & -\frac{11}{3} & -18 \\ -\frac{11}{3} & 3 & 12 \end{array}\right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} \frac{157}{9} & -\frac{11}{3} \\ -\frac{11}{3} & 3 \end{vmatrix} = \frac{350}{9} \qquad |D_1| = \begin{vmatrix} -18 & -\frac{11}{3} \\ 12 & 3 \end{vmatrix} = -10 \qquad |D_2| = \begin{vmatrix} \frac{157}{9} & -18 \\ -\frac{11}{3} & 12 \end{vmatrix} = \frac{430}{3}$$

$$x = \frac{|D_1|}{|D|} = \frac{-10}{\frac{350}{9}} = -\frac{9}{35}$$

$$y = \frac{|D_2|}{|D|} = \frac{\frac{430}{3}}{\frac{350}{9}} = \frac{129}{35}$$

Dosazení:

$$-\frac{9}{35} + \frac{129}{35} = \frac{24}{7}$$

$$-\frac{2}{3} \cdot (-\frac{9}{35}) + \frac{129}{35} = \frac{27}{7}$$

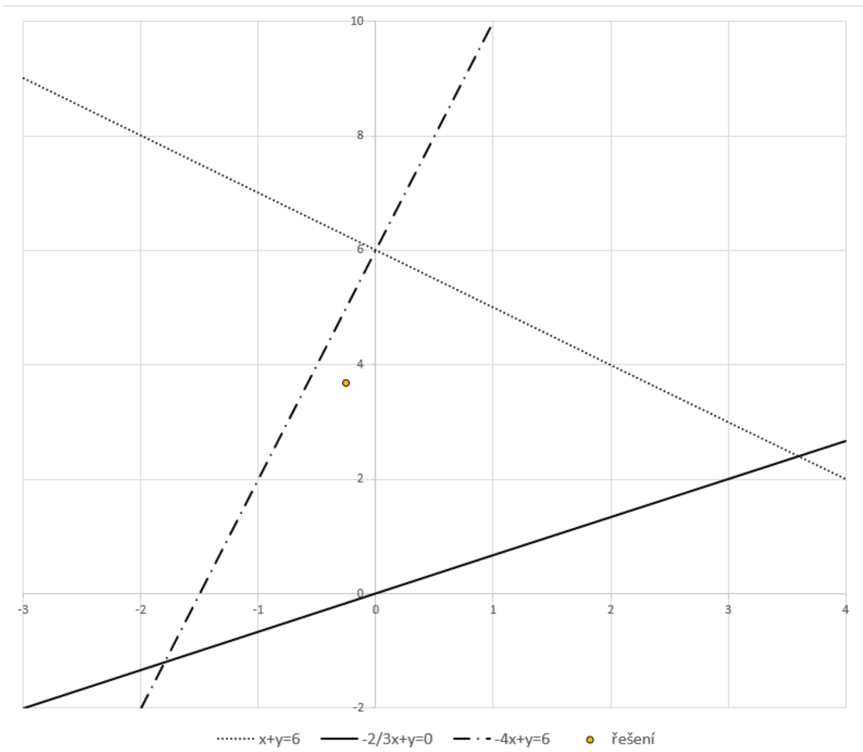
$$-4 \cdot (-\frac{9}{35}) + \frac{129}{35} = \frac{33}{7}$$

$$\varphi = (6; 0; 6)$$
$$\varphi^* = \left(\frac{24}{7}; \frac{27}{7}; \frac{33}{7}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(6 - \frac{24}{7}\right)^2 + \left(0 - \frac{27}{7}\right)^2 + \left(6 - \frac{33}{7}\right)^2} = 4,8107$$

Graf



Příklad 9.

Mějme soustavu rovnic:

$$\begin{aligned} -3x + y &= 9 \\ x + y &= 5 \\ -x + y &= 0 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 9 \\ 5 \\ 0 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 9 \\ 5 \\ 0 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = 11 \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = -3 \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = -22$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = -3 \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = 14$$

$$\left[\begin{array}{cc|c} 11 & -3 & -22 \\ -3 & 3 & 14 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} 11 & -3 \\ -3 & 3 \end{vmatrix} = 24 \qquad |D_1| = \begin{vmatrix} -22 & -3 \\ 14 & 3 \end{vmatrix} = -24 \qquad |D_2| = \begin{vmatrix} 11 & -22 \\ -3 & 14 \end{vmatrix} = 88$$

$$x = \frac{|D_1|}{|D|} = \frac{-24}{24} = -1$$

$$y = \frac{|D_2|}{|D|} = \frac{88}{24} = \frac{11}{3}$$

Dosazení:

$$-3 \cdot (-1) + \frac{11}{3} = \frac{20}{3}$$

$$-1 + \frac{11}{3} = \frac{8}{3}$$

$$-(-1) + \frac{11}{3} = \frac{14}{3}$$

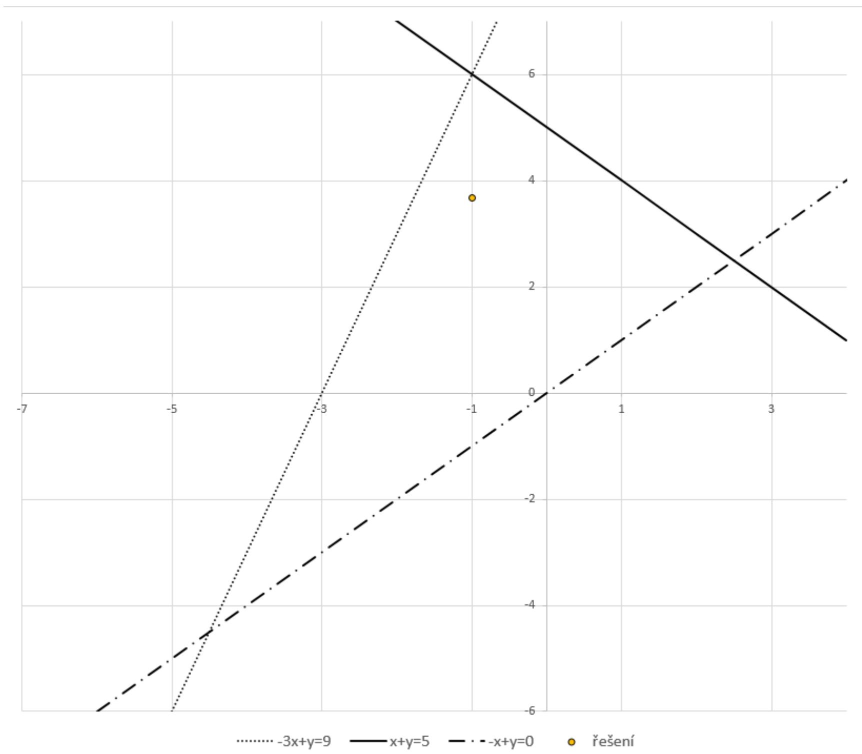
$$\varphi = (9; 5; 0)$$

$$\varphi^* = \left(\frac{20}{3}; \frac{8}{3}; \frac{14}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(9 - \frac{20}{3}\right)^2 + \left(5 - \frac{8}{3}\right)^2 + \left(0 - \frac{14}{3}\right)^2} = 5,7155$$

Graf



Příklad 10.

Mějme soustavu rovnic:

$$\begin{aligned} -2x + y &= -8 \\ -x - y &= 4 \\ \frac{1}{2}x - y &= 1 \end{aligned}$$

1. Nalezení normální soustavy:

$$\begin{bmatrix} -2 \\ -1 \\ \frac{1}{2} \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} y = \begin{bmatrix} -8 \\ 4 \\ 1 \end{bmatrix}$$

$$\varphi^{(1)} = \begin{bmatrix} -2 \\ -1 \\ \frac{1}{2} \end{bmatrix} \quad \varphi^{(2)} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \quad \varphi = \begin{bmatrix} -8 \\ 4 \\ 1 \end{bmatrix}$$

$$\left\langle \varphi^{(1)}; \varphi^{(1)} \right\rangle = \frac{21}{4} \quad \left\langle \varphi^{(1)}; \varphi^{(2)} \right\rangle = -\frac{3}{2} \quad \left\langle \varphi^{(1)}; \varphi \right\rangle = \frac{25}{2}$$

$$\left\langle \varphi^{(2)}; \varphi^{(1)} \right\rangle = -\frac{3}{2} \quad \left\langle \varphi^{(2)}; \varphi^{(2)} \right\rangle = 3 \quad \left\langle \varphi^{(2)}; \varphi \right\rangle = -13$$

$$\left[\begin{array}{cc|c} \frac{21}{4} & -\frac{3}{2} & \frac{25}{2} \\ -\frac{3}{2} & 3 & -13 \end{array} \right]$$

Využijeme cramarova pravidla a nalezneme x a y:

$$|D| = \begin{vmatrix} \frac{21}{4} & -\frac{3}{2} \\ -\frac{3}{2} & 3 \end{vmatrix} = \frac{27}{2} \qquad |D_1| = \begin{vmatrix} \frac{25}{2} & -\frac{3}{2} \\ -13 & 3 \end{vmatrix} = 18 \qquad |D_2| = \begin{vmatrix} \frac{21}{4} & \frac{25}{2} \\ -\frac{3}{2} & -13 \end{vmatrix} = -\frac{99}{2}$$

$$x = \frac{|D_1|}{|D|} = \frac{18}{\frac{27}{2}} = \frac{4}{3}$$

$$y = \frac{|D_2|}{|D|} = \frac{-\frac{99}{2}}{\frac{27}{2}} = -\frac{11}{3}$$

Dosazení:

$$-2 \cdot \frac{4}{3} + (-\frac{11}{3}) = -\frac{19}{3}$$

$$-\frac{4}{3} - (-\frac{11}{3}) = \frac{7}{3}$$

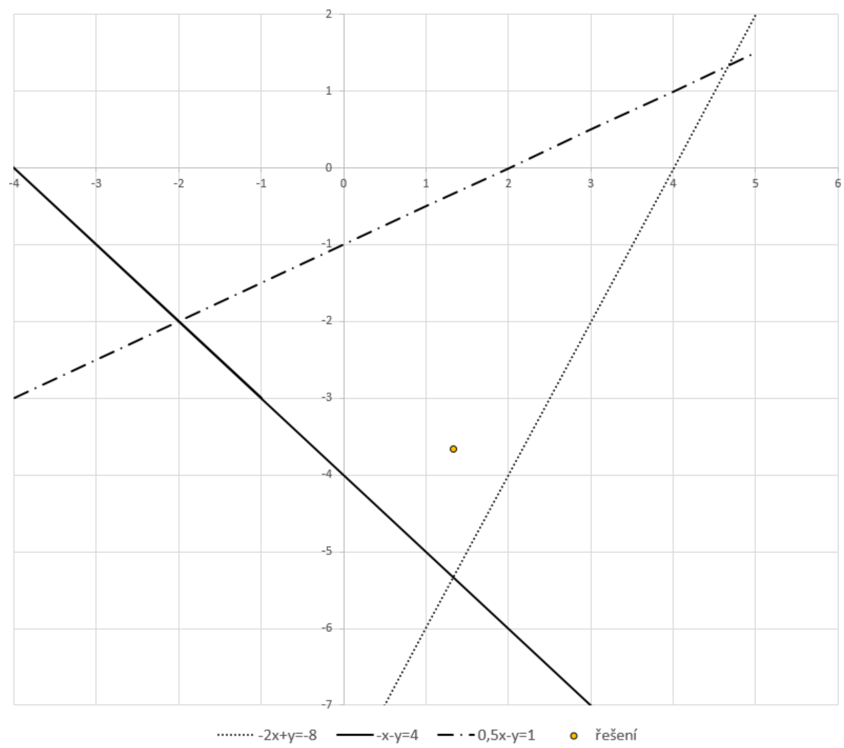
$$\frac{1}{2} \cdot \frac{4}{3} - (-\frac{11}{3}) = \frac{13}{3}$$

$$\varphi = (-8; 4; 1)$$
$$\varphi^* = \left(-\frac{19}{3}; \frac{7}{3}; \frac{13}{3}\right)$$

Odchylka:

$$||\varphi - \varphi^*|| = \sqrt{\left(-8 + \frac{19}{3}\right)^2 + \left(4 - \frac{7}{3}\right)^2 + \left(1 - \frac{13}{3}\right)^2} = 4,0825$$

Graf



Použité materiály

Mgr. HINTERLEITNER Irena , Ph.D.; RNDr. PŘIBYL Oto: MATEMATIKA I, Vybrané kapitoly z numerických výpočtů

Za spolupráce a pod vedením Mgr. Ireny Hinterleitner, Ph.D., které tímto děkuji.

V Brně 2017