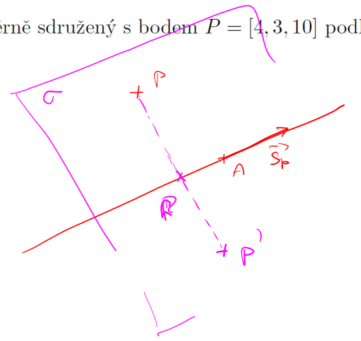
7. Určete bod souměrně sdružený s bodem $P = [4, 3, 10]$ podle přímky $p: \frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{5}$.



$p: x = 1 + 2t,\; y = 2 + 4t,\; z = 3 + 5t,\quad t \in \mathbb{R}$

$\sigma \perp p, \; \sigma \ni P$

$R = p \cap \sigma$

i)

$\sigma: \vec{n}_\sigma = \vec{s}_p = (2, 4, 5)$

$\sigma: 2x + 4y + 5z + d = 0$

$P \in \sigma: 4\cdot 2 + 3\cdot 4 + 10\cdot 5 + d = 0$, $d = -70$

$\sigma: 2x + 4y + 5z - 70 = 0$

$R = \sigma \cap p: \; 2(1+2t) + 4(2+4t) + 5(3+5t) - 70 = 0$

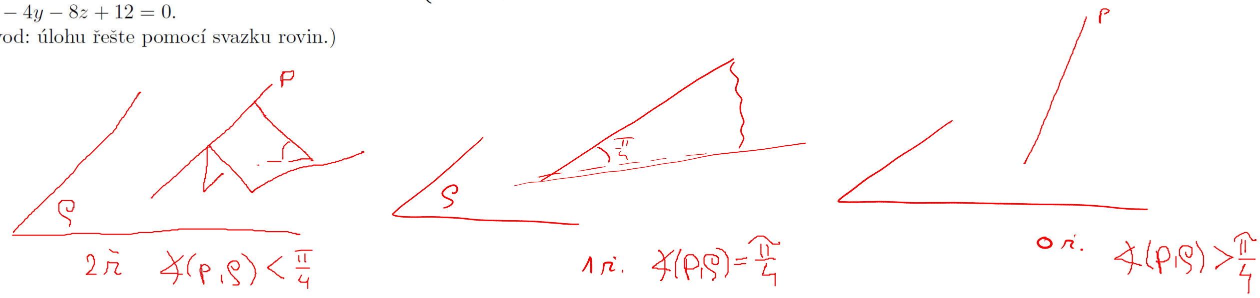
$45t - 45 = 0 \Rightarrow 45t = 45 \Rightarrow t = 1$

$R = [3, 6, 8]$

$R = \frac{P + P'}{2} \Rightarrow P' = 2R - P = 2[3,6,8] - [4,3,10] = [2, 9, 6]$

ii) $\vec{PR} \cdot \vec{s}_q = 0$; $P' = 2R - P$

10. Určete rovnici roviny, která prochází přímkou $p : \begin{cases} x + 5y + z = 0 \\ x - z + 4 = 0 \end{cases}$ a svírá úhel $\omega = \pi/4$ s rovinou $\rho : x - 4y - 8z + 12 = 0$.
(Návod: úlohu řešte pomocí svazku rovin.)



$\sigma: \ \alpha(x+5y+z) + \beta(x-z+4) = 0$

$(\alpha+\beta)x + 5\alpha y + (\alpha-\beta)z + 4\beta = 0 \quad \Rightarrow \vec{n}_\sigma = (\alpha+\beta, 5\alpha, \alpha-\beta)$

$\cos\varphi = \dfrac{|\vec{n}_\rho \cdot \vec{n}_\sigma|}{\|\vec{n}_\rho\| \cdot \|\vec{n}_\sigma\|} \ ; \quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\dfrac{|(\alpha+\beta, 5\alpha, \alpha-\beta)(1,-4,-8)|}{\sqrt{(\alpha+\beta)^2 + 25\alpha^2 + (\alpha-\beta)^2}\sqrt{1+16+64}} = \dfrac{|\alpha+\beta-20\alpha-8\alpha+8\beta|}{\sqrt{\alpha^2+2\alpha\beta+\beta^2+25\alpha^2+\alpha^2-2\alpha\beta+\beta^2}\ \sqrt{81}} =$

$= \dfrac{|-27\alpha+9\beta|}{\sqrt{27\alpha^2+2\beta^2}\cdot 9} = \dfrac{|-3\alpha+\beta|}{\sqrt{27\alpha^2+2\beta^2}} = \dfrac{\sqrt{2}}{2}$

$2|-3\alpha+\beta| = \sqrt{2}\cdot\sqrt{27\alpha^2+2\beta^2} \quad /^2$ – zde ekviv. úprava, neboť všude jsou kladná čísla

$4(9\alpha^2-6\alpha\beta+\beta^2) = 2(27\alpha^2+2\beta^2) \quad /:2$

$2(9\alpha^2-6\alpha\beta+\beta^2) = 27\alpha^2+2\beta^2$

$-9\alpha^2-12\alpha\beta = 0 \quad /\cdot(-1)$

$9\alpha^2+12\alpha\beta = 0$

$\alpha(9\alpha+12\beta) = 0 \quad \Longrightarrow$

1. $\alpha = 0,\ \beta \in \mathbb{R}$ lib. $\Rightarrow \sigma_1: \ x - z + 4 = 0$

2. $\alpha \neq 0 \Rightarrow 9\alpha + 12\beta = 0$, tj. $3\alpha + 4\beta = 0$

např. $\alpha = 4,\ \beta = -3$

$\sigma_2: 4(x+5y+z) - 3(x-z+4) = 0$

$x + 20y + 7z - 12 = 0$

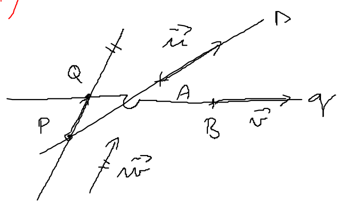
12. Určete příčku mimoběžek $p = [A, \vec{u}]$, $q = [B, \vec{v}]$, která má směr $[\vec{w}]$. Příčku r určete body $P = p \cap r, Q = q \cap r$. Přitom je $A = [-1, 1, -5]$, $\vec{u} = (1, 1, 2)$, $B = [1, -2, 3]$, $\vec{v} = (1, 3, -1)$, $\vec{w} = (1, -2, 3)$.

$\vec{u} = (1,1,2)$
$\vec{v} = (1,3,-1)$
$\vec{AB} = (2,-3,8)$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & -1 \\ 2 & -3 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 0 & -5 & 4 \end{vmatrix} = 1\cdot 1 \begin{vmatrix} 2 & -3 \\ -5 & 4 \end{vmatrix} = 8 - 15 \neq 0$$ jsou mimoběžné

$[\vec{u}, \vec{v}, \vec{AB}] = [\vec{s}_p, \vec{s}_q, \vec{AB}] = -7$

I.)



$p: \; x = -1 + t, \; y = 1 + t, \; z = -5 + 2t$

$q: \; x = 1 + s, \; y = -2 + 3s, \; z = 3 - s$

$P: [-1+t, 1+t, -5+2t]$ $\quad Q[1+s, -2+3s, 3-s]$

$\Rightarrow \vec{PQ} = [2+s-t, -3+3s-t, 8-s-2t]$

$\vec{PQ} = k\,\vec{w} \Rightarrow$

$2+s-t = k$
$-3+3s-t = -2k$
$8-s-2t = 3k$

$1+5s-3t = 0$
$2-4s+t = 0 \; / \cdot 3$

$7 - 7s = 0$
$s = 1$
$t = 2$
$(k = 1)$

$P = [1,3,-1]$, $Q[2,1,2]$

$r(P, \vec{w}): \; x = 1 + l, \; y = 3 - 2l, \; z = -1 + 3l$

II.) $\alpha(A, \vec{u}, \vec{w})$, $\beta(B, \vec{v}, \vec{w})$

$r = \alpha \cap \beta$

$\vec{n}_\alpha = \vec{u} \times \vec{w}$
$\vec{n}_\beta = \vec{v} \times \vec{w}$

12. Určete příčku mimoběžek $p = [A, \vec{u}]$, $q = [B, \vec{v}]$, která má směr $[\vec{w}]$. Příčku r určete body $P = p \cap r, Q = q \cap r$. Přitom je $A = [-1, 1, -5]$, $\vec{u} = (1, 1, 2)$, $B = [1, -2, 3]$, $\vec{v} = (1, 3, -1)$, $\vec{w} = (1, -2, 3)$.

7.6. 13. Určete osu mimoběžek a (nejkratší) vzdálenost mimoběžek z příkladu 12.

$$\vec{w} = \vec{s_p} \times \vec{s_q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 1 & 3 & -1 \end{vmatrix} = \dots = (-7, 3, 2)$$

($\vec{s_p} = \vec{u}$, $\vec{s_q} = \vec{v}$; DU)

viz 7.6.12

$$\begin{aligned} 2 + s - t &= -7k \\ -3 + 3s - t &= 3k \\ 8 - s - 2t &= 2k \end{aligned}$$

$$\vec{PQ} = k\vec{w}$$

$$\begin{pmatrix} 1 & -1 & 7 & | & -2 \\ 3 & -1 & -3 & | & 3 \\ -1 & -2 & 2 & | & -8 \end{pmatrix} \sim \text{G.E.M} \sim$$

(sloupce s, t, k; DU)

$$k = -\frac{7}{62}, \quad t = \frac{195}{62}, \quad s = \frac{120}{62}$$

$$P = [-1 + t, 1 + t, -5 + 2t] = \left[\frac{133}{62}, \frac{257}{62}, \frac{80}{62}\right]$$

$$Q = [1 + s, -2 + 3s, 3 - s] = \left[\frac{182}{62}, \frac{236}{62}, \frac{66}{62}\right]$$

$$v(p, q) = \|\vec{PQ}\| = \dots$$

$$v(p, q) = \frac{|[\vec{s_p}, \vec{s_q}, \vec{AB}]|}{\|\vec{s_p} \times \vec{s_q}\|} = \frac{|-7|}{\sqrt{49 + 9 + 4}} = \frac{7}{\sqrt{62}}$$