Příklad 10.1

Vypočítejte limity posloupností

a) $\lim\limits_{n\to\infty} \dfrac{1+(-1)^n}{2}$,

b) $\lim\limits_{n\to\infty} \dfrac{3n^2-123n-1000}{2n^2+n}$,

c) $\lim\limits_{n\to\infty} (5n^2-3n)$,

d) $\lim\limits_{n\to\infty} \dfrac{-n^2}{100n+2}$,

e) $\lim\limits_{n\to\infty} \sqrt{n+1}(\sqrt{2n-1}-\sqrt{2n+1})$.

a) $\lim\limits_{n\to\infty} \dfrac{1+(-1)^n}{2}$ = neexistuje

$n=1 \to 0$
$n=2 \to 1$
$n=3 \to 0$
$n=4 \to 1$
$\vdots \qquad \vdots$

b) $$\lim_{n\to\infty} \frac{3n^2-123n-1000}{2n^2+n} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(3-\overset{\to 0}{\frac{123}{n}}-\overset{\to 0}{\frac{1000}{n^2}}\right)}{\cancel{n^2}\left(2+\underset{\to 0}{\frac{1}{n}}\right)} = \frac{3}{2}$$

c) $$\lim_{n\to\infty} (5n^2-3n) \; |\infty-\infty| = \lim_{n\to\infty} n\cdot(5n-3) \; |\infty\cdot\infty| = \infty\cdot\infty = \infty$$

$$\left(\lim_{n\to\infty} (-5n^2+3n) \; |-\infty+\infty| = \lim_{n\to\infty} n\,(-5n+3)\,|\infty\cdot(-\infty)| = \ldots\right.$$

d) $$\lim_{n\to\infty} \frac{-n^2}{100n+2} = \lim_{n\to\infty} \frac{-n^{\cancel{2}}}{\cancel{n}\left(100+\underset{\to 0}{\frac{2}{n}}\right)} = \frac{-\infty}{100} = -\infty$$

e) $$\lim_{n\to\infty} \sqrt{n+1}\left(\sqrt{2n-1}-\sqrt{2n+1}\right) \; |\infty\cdot(\infty-\infty)| =$$

$$= \lim_{n\to\infty} \sqrt{n+1}\,\underset{(a\;-\;b)}{\left(\sqrt{2n-1}-\sqrt{2n+1}\right)}\cdot\frac{\overset{(a\;+\;b)}{\sqrt{2n-1}+\sqrt{2n+1}}}{\sqrt{2n-1}+\sqrt{2n+1}} = \lim_{n\to\infty} \sqrt{n+1}\cdot\frac{(\cancel{2n}-1-\cancel{2n}-1)}{\sqrt{2n-1}+\sqrt{2n+1}} \left|\frac{-\infty}{\infty}\right|$$

$$= -\lim_{n\to\infty} \frac{2\cancel{\sqrt{n}}\left(\sqrt{1+\overset{\to 0}{\frac{1}{n}}}\right)}{\cancel{\sqrt{n}}\left(\sqrt{2-\underset{\to 0}{\frac{1}{n}}}+\sqrt{2+\underset{\to 0}{\frac{1}{n}}}\right)} = -\frac{\cancel{2}}{\cancel{2}\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

Příklad 10.2

Příklady posloupností, pro něž

$$\lim a_n = \lim b_n = \infty, \text{ ale } \lim(a_n - b_n) \text{ je:}$$

a) ∞ : $a_n = n^2, b_n = n,$

b) $-\infty$: $a_n = n, b_n = n^3,$

c) a : $a_n = n + 5, b_n = n + 1,$

$a_n = n^2 + \cos\frac{1}{n}, b_n = n^2.$

a) $\lim\limits_{n\to\infty} (n^2 - n) \left|\infty - \infty\right| = \lim\limits_{n\to\infty} n\cdot(n-1) \left|\infty\cdot\infty\right| = \infty$

b) $\lim\limits_{n\to\infty} (n - n^3) \left|\infty - \infty\right| = \lim\limits_{n\to\infty} n(1-n^2) \left|\infty\cdot(-\infty)\right| = -\infty$

c) $\lim\limits_{n\to\infty} (n+5 - (n+1)) = \lim\limits_{n\to\infty} 4 = \underline{\underline{4}}$

Příklad 10.3

Příklady posloupností, pro něž

$$\lim a_n = 0, \lim b_n = \infty, \text{ ale } \lim(a_n \cdot b_n) \text{ je:}$$

a) ∞ : $a_n = \frac{1}{n}, b_n = n^2,$

b) 0 $\cancel{-\infty}$: $a_n = \frac{1}{n^2}, b_n = n,$

c) a : $a_n = \frac{1}{n}, b_n = 2n,$

d) $\nexists$: $a_n = \frac{\sin n}{n^2}, b_n = n^2.$

a) $\lim\limits_{n\to\infty} \frac{n^{\cancel{2}}}{\cancel{n}} = \lim\limits_{n\to\infty} n = \underline{\underline{\infty}}$

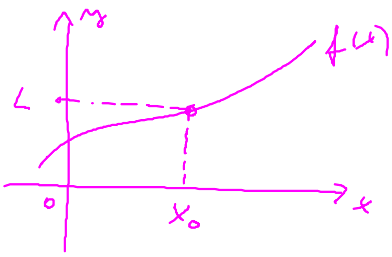
b) $\lim\limits_{n\to\infty} \frac{\cancel{n}}{n^{\cancel{2}}} = \lim\limits_{n\to\infty} \frac{1}{n} = \underline{\underline{0}}$

c) $\lim\limits_{n\to\infty} \frac{2\cancel{n}}{\cancel{n}} = \underline{\underline{2}}$

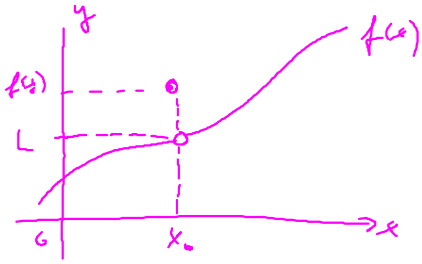
d) $\lim\limits_{n\to\infty} \frac{\cancel{n^2} \cdot \sin n}{\cancel{n^2}} = \lim\limits_{n\to\infty} \sin n = \underline{\underline{\nexists}}$

POZNÁMKA: $\lim\limits_{x \to x_0} f(x) = L$

a)

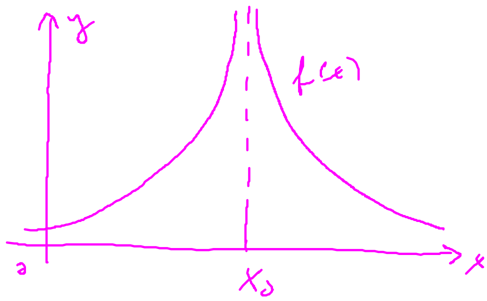


$x_0 \in \mathbb{R},\ L \in \mathbb{R}$



$x_0 \in \mathbb{R},\ L \in \mathbb{R}$

b)

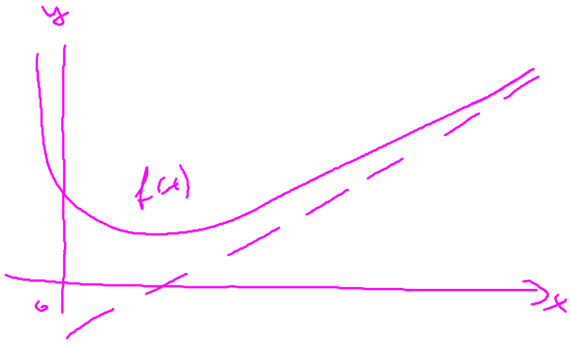


$x_0 \in \mathbb{R},\ L = \infty$

c)

$x_0 = \infty,\ L \in \mathbb{R}$

d)



$x_0 = \infty,\ L = \infty$

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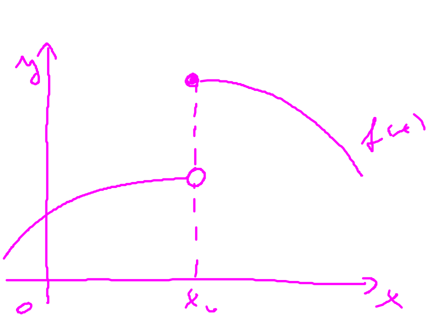
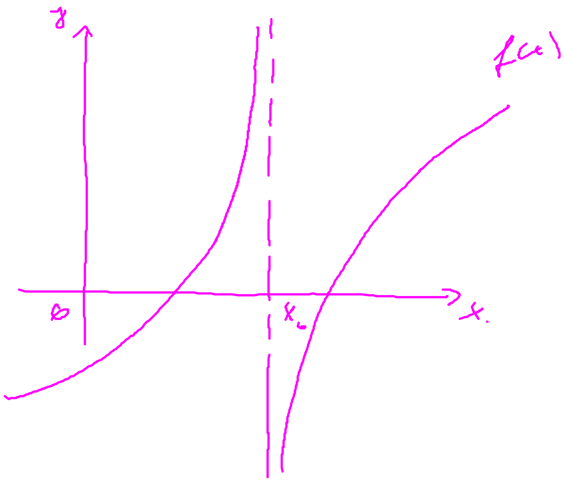
fce $f(x)$ nemá v x_0 limitu





Příklad 10.4

Vypočítejte limity ~~posloupností~~ funkcí

a) $\lim\limits_{x\to 1}(x^2+1)$,

b) $\lim\limits_{x\to\infty}(x+\left(\frac{1}{2}\right)^x)$,

c) $\lim\limits_{x\to\infty}\dfrac{x^2+2x-1}{3x^2+5x+2}$,

d) $\lim\limits_{x\to\infty}\dfrac{x+2}{x^2+5x+1}$,

e) $\lim\limits_{x\to\infty}\dfrac{x^2+5x+1}{x+2}$,

f) $\lim\limits_{x\to\infty}\ln\dfrac{x+1}{x+2}$.

a) $\lim\limits_{x\to 1}(x^2+1)=\underline{\underline{2}}$

b) $\lim\limits_{x\to\infty}\left(x+\left(\frac{1}{2}\right)^x\right)=\lim\limits_{x\to\infty}x+\lim\limits_{x\to\infty}\left(\frac{1}{2}\right)^x=\infty+0=\underline{\underline{\infty}}$

c) $\lim\limits_{x\to\infty}\dfrac{x^2+2x-1}{3x^2+5x+2}\left|\dfrac{\infty}{\infty}\right|=\lim\limits_{x\to\infty}\dfrac{\cancel{x^2}(1+\frac{2}{x}-\frac{1}{x^2})}{\cancel{x^2}(3+\frac{5}{x}+\frac{2}{x^2})}=\underline{\underline{\frac{1}{3}}}$

e) $\lim\limits_{x\to\infty}\dfrac{x^2+5x+1}{x+2}\left|\dfrac{\infty}{\infty}\right|=\lim\limits_{x\to\infty}\dfrac{\cancel{x^2}(1+\frac{5}{x}+\frac{1}{x^2})}{\cancel{x}(1+\frac{2}{x})}=\underline{\underline{\infty}}$

d) $\lim\limits_{x\to\infty}\dfrac{x+2}{x^2+5x+1}\left|\dfrac{\infty}{\infty}\right|=\lim\limits_{x\to\infty}\dfrac{\cancel{x}(1+\frac{2}{x})}{\cancel{x^2}(1+\frac{5}{x}+\frac{1}{x^2})}=\lim\limits_{x\to\infty}\frac{1}{x}=\underline{\underline{0}}$

f) $\lim\limits_{x\to\infty}\ln\left(\dfrac{x+1}{x+2}\right)=\ln\left(\lim\limits_{x\to\infty}\dfrac{x+1}{x+2}\right)=\ln 1=\underline{\underline{0}}$

Příklad 10.5

Vypočítejte limity ~~posloupností~~ funkcí

a) $\lim\limits_{x\to 1} \dfrac{x-3}{(x-1)^2}$,

b) $\lim\limits_{x\to\infty} (x^2 - x + 1)$,

c) $\lim\limits_{x\to\infty} e^{\frac{1}{x}}$,

d) $\lim\limits_{x\to 2} f(x)$, kde $f : x^2$, pro $x \in \mathbb{R} - \{2\} \wedge f(2) = 10$.

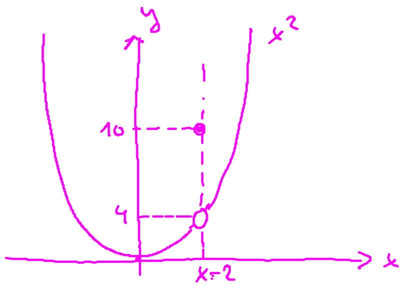
a) $\lim\limits_{x\to 1} \dfrac{x-3}{(x-1)^2} \left| \dfrac{-k}{0^+} \right| = -k \cdot \infty = \underline{\underline{-\infty}}$

$= \lim\limits_{x\to 1} (x-3) \cdot \lim\limits_{x\to 1} \dfrac{1}{(x-1)^2} = -2 \cdot \infty = \underline{\underline{-\infty}}$

b) $\lim\limits_{x\to\infty} (x^2 - x + 1) \; |\infty - \infty| = \lim\limits_{x\to\infty} x^2 \left(1 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}\right) = \underline{\underline{\infty}}$

c) $\lim\limits_{x\to\infty} e^{\frac{1}{x}} = e^{\lim\limits_{x\to\infty} \frac{1}{x}} = e^0 = \underline{\underline{1}}$

d) $\lim\limits_{x\to 2} f(x)$, $f : x^2$, $x \in \mathbb{R} - \{2\} \wedge f(2) = 10$



$\lim\limits_{x\to 2} x^2 = 4$