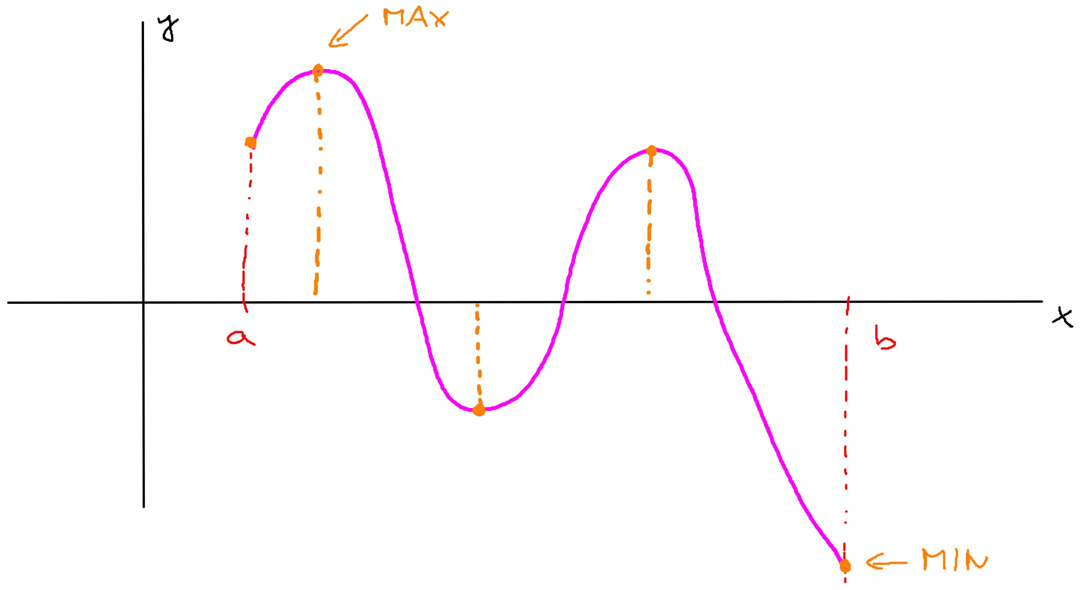
WEIERSTRASSOVA VĚTA



Příklad 13.1

Určete přibližně hodnotu $\ln \frac{5}{4}$.

$$f'(x_0) \doteq \frac{f(x)-f(x_0)}{x-x_0} \qquad f(x) \doteq f(x_0) + f'(x_0)(x-x_0)$$

$$\ln \frac{5}{4} \doteq \ln \underset{x_0}{1} + df(\underset{x_0}{1}, \underset{h}{\tfrac{1}{4}}) = \% \qquad df(x_0, h) = f'(x_0)\cdot h$$

$$f(x) = \ln x \quad ; \quad f'(x) = \frac{1}{x} \qquad f'(x_0) = f'(1) = 1$$

$$df(1, \tfrac{1}{4}) = f'(1)\cdot \frac{1}{4} = 1 \cdot \frac{1}{4} = 0{,}25$$

$$\% = \underset{0}{\underset{\shortparallel}{\ln 1}} + 0{,}25 = 0{,}25$$

Příklad 13.2

Vypočítejte $\mathrm{d}^3 f(2, 0.1)$, jestliže $f(x) = \operatorname{arctg} x$.

$$d^3 f(2; 0{,}1) = f'''(2) \cdot 0{,}1^3$$

$$f(x) = \operatorname{arctg} x$$

$$f'(x) = \frac{1}{1+x^2} = (1+x^2)^{-1}$$

$$f''(x) = (-1)(1+x^2)^{-2} \cdot 2x = -\frac{2x}{(1+x^2)^2}$$

$$f'''(x) = -\frac{2(1+x^2)^2 - 2x \cdot 2 \cdot (1+x^2) \cdot 2x}{(1+x^2)^4} = -\frac{\cancel{(1+x^2)}(2+2x^2-8x^2)}{(1+x^2)^{\cancel{4}\,3}} = \frac{6x^2-2}{(1+x^2)^3}$$

$$f'''(2) = \frac{6 \cdot 2^2 - 2}{(1+2^2)^3} = \frac{22}{125}$$

$$d^3 f(2; 0{,}1) = \frac{22}{125} \cdot \underbrace{0{,}1^3}_{= \frac{1}{1000}} = \underline{\underline{\frac{22}{125000}}}$$

Příklad 13.3

Určete Taylorův (Maclaurinův) polynom funkce f v bodě x_0 stupně n, je-li

a) $f(x) = \operatorname{tg} x, x = 0, n = 1,$

b) $f(x) = x\sqrt{x}, x = 1, n = 3,$

c) $f(x) = \sin x, x = 0, n = 4.$

a) $f(x) = \operatorname{tg} x \;,\; x_0 = 0 \;,\; n = 1$

$$T_n(f, x_0) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \dots + \frac{f^{n}(x_0)}{n!}(x-x_0)^n$$

$$f(0) = \operatorname{tg} 0 = 0$$

$$f' = \frac{1}{\cos^2 x} \;;\; f'(0) = \frac{1}{\cos^2 0} = 1$$

$$T_1 = 0 + 1 \cdot x = \underline{\underline{x}}$$

$$= 0 + \frac{1}{1!}(x-0)$$

$$R_n(x) = \frac{f^{n+1}(\xi)}{(n+1)!}(x-x_0)^{n+1} \;,\; \xi = x_0 + t(x-x_0) \;,\; 0 < t < 1$$

$$f(x) = T_1(x) + R_1(x) \qquad \xi = 0 + tx = tx$$

$$f'' = -2 \cdot \cos^{-3} x \cdot (-\sin x) = \frac{2\sin x}{\cos^3 x} \qquad f''(tx) = \frac{2\sin(tx)}{\cos^3(tx)}$$

$$f(x) = 0 + 1 \cdot x + \frac{\cancel{2}\sin(tx)}{\cancel{2}\cdot\cos^3(tx)}(x-0)^2 = \underline{x + \frac{\sin(tx)}{\cos^3(tx)} x^2} \;;\; 0 < t < 1$$

b) $f(x) = x\sqrt{x} \;,\; x_0 = 1 \;,\; n = 3$

$$f(x) = x^{\frac{3}{2}} \qquad f(1) = 1$$

$$f'(x) = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x} \qquad f'(1) = \frac{3}{2}$$

$$f''(x) = \frac{3}{2} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{3}{4} \cdot x^{-\frac{1}{2}} = \frac{3}{4\sqrt{x}} \qquad f''(1) = \frac{3}{4}$$

$$f'''(x) = \frac{3}{4}\left(-\frac{1}{2}\right) x^{-\frac{3}{2}} = -\frac{3}{8\sqrt{x^3}} \qquad f'''(1) = -\frac{3}{8}$$

$$T_3(x) = 1 + \frac{\frac{3}{2}}{1!}(x-1) + \frac{\frac{3}{4}}{2!}(x-1)^2 + \frac{-\frac{3}{8}}{3!}(x-1)^3$$

$$= \underline{\underline{1 + \frac{3}{2}(x-1) + \frac{3}{8}(x-1)^2 - \frac{1}{16}(x-1)^3}}$$

c) $f(x) = \sin x$, $x_0 = 0$, $n = 4$

$$f(0) = 0$$

$$f' = \cos x \qquad f'(0) = 1$$

$$f'' = -\sin x \qquad f''(0) = 0$$

$$f''' = -\cos x \qquad f'''(0) = -1$$

$$f^{IV} = \sin x \qquad f^{IV}(0) = 0$$

$$T_4 = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \frac{f^{IV}(x_0)}{4!}(x-x_0)^4$$

$$T_4 = 0 + \frac{1}{1!}(x-0) + \cancel{\frac{0}{2!}(x-0)^2} + \frac{(-1)}{3!}(x-\cancel{0})^3 + \cancel{\frac{0}{4!}(x-0)^4}$$

neuvádím!

$$= x - \frac{x^3}{6}$$

Příklad 13.4

Vypočtěte derivaci funkce dané parametricky rovnicemi

$$f: \begin{cases} x = \cos t \\ y = \sin t \end{cases}.$$

$$y'_x = \frac{y'_t}{x'_t} \;;\quad y''_{xx} = \frac{(y'_x)'_t}{x'_t} \;;\quad y'''_{xxx} = \frac{(y''_{xx})'_t}{x'_t}$$

$$f: \begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad \begin{aligned} x'_t &= -\sin t \\ y'_t &= \cos t \end{aligned}$$

$$y'_x = \frac{y'_t}{x'_t} = -\frac{\cos t}{\sin t} = -\cot g\, t$$

$$y''_{xx} = \frac{(y'_x)'_t}{x'_t} = \frac{\frac{1}{\sin^2 t}}{-\sin t} = -\frac{1}{\sin^3 t} = -\sin^{-3} t$$

$$y'''_{xxx} = \frac{(y''_{xx})'_t}{x'_t} = \frac{-(-3)\sin^{-4} t \cdot \cos t}{-\sin t} = -\frac{3\cos t}{\sin^5 t}$$