6. Vlastní čísla a vlastní vektory matice

Příklad 6.1. Najděte vlastní čísla a vlastní vektory matice:

c) $\begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, **d)** $\begin{pmatrix} 2 & 1 & -3 \\ 3 & -2 & -3 \\ 1 & 1 & -2 \end{pmatrix}$, **e)** $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}$,

f) $\begin{pmatrix} 1 & 10 & 3 \\ 2 & 1 & 2 \\ 3 & 10 & 1 \end{pmatrix}$, **g)** $\begin{pmatrix} -1 & 4 & 3 \\ -2 & 5 & 3 \\ 2 & -4 & -2 \end{pmatrix}$, **h)** $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$,

i) $\begin{pmatrix} 0 & -2 & -2 \\ -1 & 1 & 2 \\ -1 & -1 & 2 \end{pmatrix}$, **j)** $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$,

Výsledek:

c) $\begin{aligned} \lambda_1 &= 1, (-s, 0, 2)^T \\ \lambda_2 &= 2, (t, t, -t)^T \qquad s, t, u \in \mathbb{R} \\ \lambda_3 &= 3, (0, -u, u)^T \end{aligned}$

d) $\begin{aligned} \lambda_1 &= 1, (2s, s, s)^T \\ \lambda_2 &= -1, (t, 0, -t)^T \qquad s, t, u \in \mathbb{R} \\ \lambda_3 &= -2, (u, -u, u)^T \end{aligned}$

e) $\begin{aligned} \lambda_1 &= -2, (s, 0, -s)^T \\ \lambda_2 &= 1, (0, t, 0)^T \qquad s, t, u \in \mathbb{R} \\ \lambda_3 &= 4, (3u, 4u, 3u)^T \end{aligned}$

f) $\begin{aligned} \lambda_1 &= -2, (-s, 0, s)^T \\ \lambda_2 &= 9, (2t, t, 2t)^T \qquad s, t, u \in \mathbb{R} \\ \lambda_3 &= -4, (5u, -4u, 5u)^T \end{aligned}$

g) $\begin{aligned} \lambda_1 &= 0, (s, s, -s)^T \\ \lambda_{2,3} &= 1, (2u + 3t, u, 2t)^T \qquad s, t, u \in \mathbb{R} \end{aligned}$

h) $\begin{aligned} \lambda_1 &= 8, (2s, s, 2s)^T \\ \lambda_{2,3} &= -1, (u, 2t, -u - t)^T \qquad s, t, u \in \mathbb{R} \end{aligned}$

i) $\begin{aligned} \lambda_1 &= -1, (8s, s, 3s)^T \\ \lambda_{2,3} &= 2, (-t, t, 0)^T \qquad s, t \in \mathbb{R} \end{aligned}$

j) $\begin{aligned} \lambda_1 &= 5, (s, s, s)^T \\ \lambda_{2,3} &= 1, (-2u - t, u, t)^T \qquad s, t, u \in \mathbb{R} \end{aligned}$