CVIČENÍ č. 1

$$\int f(x)\,dx = F(x) \iff F'(x) = f(x),\quad F(x) - \text{PRIMITIVNÍ FCE K FCI } f(x)$$

$$F(x);\ F(x)+c,\quad c\in\mathbb{R}$$

$$\int c\cdot f(x)\,dx = c\cdot\int f(x)\,dx$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

1) $$\int \left(x + \frac{1}{x} + \sqrt{x} + \frac{1}{\sqrt{x}}\right)dx = \int x\,dx + \int \frac{1}{x}\,dx + \int x^{\frac{1}{2}}\,dx + \int x^{-\frac{1}{2}}\,dx =$$

$$= \frac{x^2}{2} + \ln|x| + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \underline{\underline{\frac{x^2}{2} + \ln|x| + \frac{2}{3}\sqrt{x^3} + 2\sqrt{x} + c}}$$

2) $$\int \frac{5\sin^2 x + 3\cos^2 x}{2\sin^2 x\cos^2 x}\,dx = \int \frac{5\cancel{\sin^2 x}}{2\cancel{\sin^2 x}\cos^2 x}\,dx + \int \frac{3\cancel{\cos^2 x}}{2\sin^2 x\cancel{\cos^2 x}}\,dx =$$

$$= \frac{5}{2}\int \frac{1}{\cos^2 x}\,dx + \frac{3}{2}\int \frac{1}{\sin^2 x}\,dx = \underline{\underline{\frac{5}{2}\operatorname{tg} x - \frac{3}{2}\operatorname{cotg} x + c}}$$

3) $$\int \cos^2 \frac{x}{2}\,dx = \int \frac{1+\cos x}{2}\,dx = \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \cos x\,dx =$$

$$\frac{x}{2} \equiv \tilde{x}$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \sin^2 x + \cos^2 x = 1$$

$$\cos^2 \tilde{x} = \frac{1+\cos 2\tilde{x}}{2} \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$= \frac{1}{2}x + \frac{1}{2}\sin x + c = \underline{\underline{\frac{1}{2}(x + \sin x) + c}}$$

4) $$\int \frac{e^{2x}-1}{e^x-1}\,dx = \int \frac{(e^x)^2-1}{e^x-1}\,dx = \int \frac{\cancel{(e^x-1)}(e^x+1)}{\cancel{e^x-1}}\,dx = \int e^x+1\,dx =$$

$$e^{a\cdot b} = (e^a)^b\ ;\quad a^2-b^2 = (a-b)(a+b)$$

$$= \underline{\underline{e^x + x + c}}$$

5) $\int \frac{x^2}{x^2+1}\,dx = \int \frac{x^2+1-1}{x^2+1}\,dx = \int \underbrace{\cancel{\frac{x^2+1}{x^2+1}}}_{1}\,dx - \int \frac{1}{x^2+1}\,dx = \int dx - \int \frac{dx}{x^2+1} =$

$= \underline{\underline{x - \operatorname{arctg} x + C}}$

I. SUBSTITUČNÍ METODA : $f(g(x)) \to t = g(x)$

II. SUBSTITUČNÍ METODA : $f(x) \to x = \varphi(t)$

6) $\int \frac{x+2}{2x+1}\,dx = \int \frac{\frac{1}{2}(2x+4)}{2x+1}\,dx = \frac{1}{2}\int \frac{2x+1+3}{2x+1}\,dx = \frac{1}{2}\left(\int \cancel{\frac{2x+1}{2x+1}}\,dx + \int \frac{3}{2x+1}\,dx\right) =$

$= \frac{1}{2}\left(\int dx + 3\underbrace{\int \frac{1}{2x+1}\,dx}_{I}\right) = \underline{\underline{\frac{1}{2}\left(x + \frac{3}{2}\ln|2x+1|\right) + c}} = \underline{\underline{\frac{x}{2} + \frac{3}{4}\ln|2x+1| + c}}$

$I = \int \frac{dx}{2x+1} \quad \left| \begin{array}{l} t = 2x+1 \\ dt = 2\,dx \Rightarrow dx = \frac{dt}{2} = \frac{1}{2}dt \end{array} \right| = \int \frac{1}{t}\,\frac{1}{2}\,dt = \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| =$

$= \frac{1}{2}\ln|2x+1|$

$\int \frac{1}{ax+b}\,dx = \frac{\ln|ax+b|}{a}$; $\int f(ax+b)\,dx = \frac{F(ax+b)}{a}$

7) $\int \frac{5}{\sqrt{2-49x^2}}\,dx = 5\int \frac{1}{\sqrt{2}\cdot\sqrt{1-\frac{49x^2}{2}}}\,dx = \frac{5}{\sqrt{2}}\int \frac{1}{\sqrt{1-(\underbrace{\tfrac{7}{\sqrt{2}}x}_{\tilde{x}})^2}}\,dx \quad \left| \begin{array}{l} t = \frac{7}{\sqrt{2}}x \\ dt = \frac{7}{\sqrt{2}}dx \Rightarrow dx = \frac{\sqrt{2}}{7}dt \end{array} \right|$

$\int \frac{1}{\sqrt{1-\tilde{x}^2}}\,dx = \arcsin x$

$= \frac{5}{\cancel{\sqrt{2}}}\cdot\frac{\cancel{\sqrt{2}}}{7}\int \frac{1}{\sqrt{1-t^2}}\,dt = \frac{5}{7}\arcsin t = \underline{\underline{\frac{5}{7}\arcsin\left(\frac{7}{\sqrt{2}}x\right) + c}}$

8) $\int \frac{1}{x\cdot \ln x\cdot \ln(\ln x)}\,dx \quad \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}dx \end{array} \right| = \int \frac{1}{t\,\ln t}\,dt \quad \left| \begin{array}{l} s = \ln t \\ ds = \frac{1}{t}dt \end{array} \right| =$

$= \int \frac{1}{s}\,ds = \ln|s| = \ln|\ln t| = \underline{\underline{\ln|\ln(\ln x)| + c}}$

9) $\int \frac{1}{\underbrace{x^2+4x}+29}\,dx = \int \frac{1}{(x+2)^2-4+29}\,dx = \int \frac{1}{25+(x+2)^2}\,dx = \frac{1}{25}\int \frac{1}{1+\frac{1}{25}(x+2)^2}\,dx$

$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$

$$= \frac{1}{25}\int \frac{1}{1+\left(\frac{x+2}{5}\right)^2}dx \left| \begin{array}{l} t = \frac{1}{5}(x+2) \\ dt = \frac{1}{5}dx \Rightarrow dx = 5dt \end{array} \right| = \frac{1}{25}\cdot 5\int \frac{1}{1+t^2}dt =$$

$$= \frac{1}{5}\operatorname{arctg} t = \underline{\underline{\frac{1}{5}\operatorname{arctg}\left(\frac{x+2}{5}\right) + c}}$$

10) $\int \frac{1}{\sqrt{3+2x-x^2}}dx$ DÚ

DOPLNIT NA ČTVEREC $\rightsquigarrow$ arcsin $\left[\arcsin\frac{x-1}{2} + c\right]$

11) $$\int (x^2-x+2)\cdot e^{3x}dx = \left| \begin{array}{ll} u = x^2-x+2 & v' = e^{3x} \\ u' = 2x-1 & v = \frac{e^{3x}}{3} \end{array} \quad \int f(ax+b) = \frac{F(ax+b)}{a} \right|$$

PER PARTES

$$\int [u(x)\cdot v(x)]' = \int (u'(x)\, v(x) + u(x)\cdot v'(x))$$

$$u(x)\cdot v(x) = \int u'(x)\, v(x)\, dx + \int u(x)\, v'(x)\, dx$$

$$\underline{\int u(x)\cdot v'(x)\, dx = u(x)\cdot v(x) - \int u'(x)\, v(x)\, dx}$$

$$= (x^2-x+2)\frac{e^{3x}}{3} - \frac{1}{3}\int (2x-1)e^{3x}dx \left| \begin{array}{ll} u = 2x-1 & v' = e^{3x} \\ u' = 2 & v = \frac{e^{3x}}{3} \end{array} \right| =$$

$$= \frac{1}{3}(x^2-x+2)e^{3x} - \frac{1}{3}\left(\frac{1}{3}(2x-1)\cdot e^{3x} - \frac{2}{3}\int e^{3x}dx\right) =$$

$$= \underline{\underline{\frac{1}{3}(x^2-x+2)e^{3x} - \frac{1}{9}(2x-1)e^{3x} + \frac{2}{27}e^{3x} + c}}$$