



FACULTY OF CIVIL institute
ENGINEERING of mathematics
and descriptive geometry

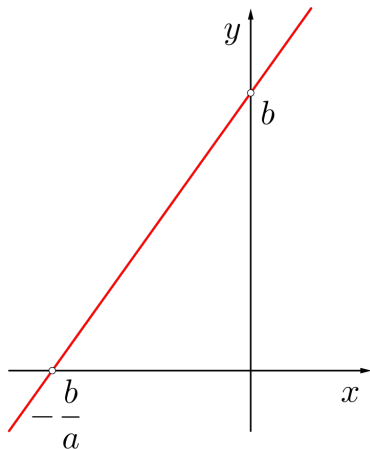
Elementární funkce

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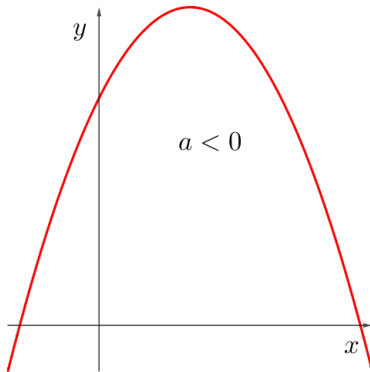
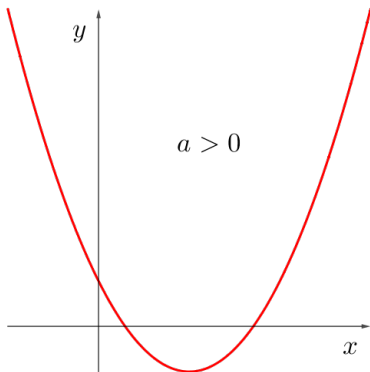
Lineární funkce $y = ax + b$

- $a, b \in \mathbb{R}; D(f) = \mathbb{R}$
- graf: přímka
- $a = 0$: **konstantní funkce**
 $y = b$
- $b = 0$: **přímá úměrnost**
 $y = ax$



Kvadratická funkce $y = ax^2 + bx + c$

- $a, b, c \in \mathbb{R}, a \neq 0, D(f) = \mathbb{R}$
- graf: parabola



Lineární lomená funkce $y = \frac{ax + b}{cx + d}$

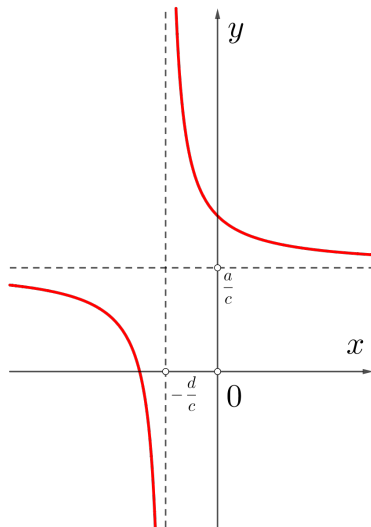
- $a, b, c, d \in \mathbb{R}; c \neq 0; ad - bc \neq 0;$

$$D(f) = \mathbb{R} - \left\{ -\frac{d}{c} \right\}$$

- graf: rovnoosá hyperbola

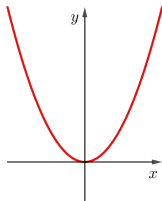
- zvláštní případ: **nepřímá**

úměrnost $y = \frac{k}{x}, k \in \mathbb{R} - \{0\}$

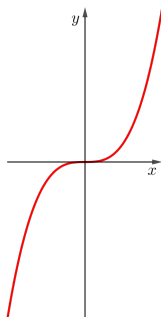


Mocninná funkce $y = x^n$

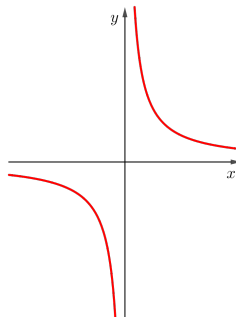
- $n \in \mathbb{N}$, $D(f) = \mathbb{R}$
- graf: parabola n -tého stupně



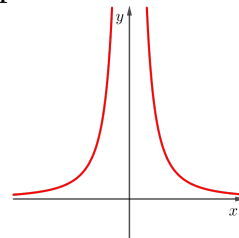
$n \in \mathbb{N}$
 n – sudé



$n \in \mathbb{N}$
 n – liché



$n \in \mathbb{Z}^-$
 n – liché

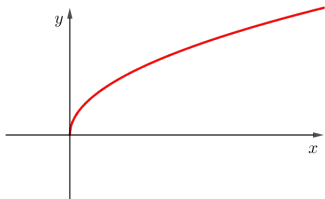


$n \in \mathbb{Z}^-$
 n – sudé

- $n \in \mathbb{Z}^-$, $D(f) = \mathbb{R} - \{0\}$
- graf: hyperbola n -tého stupně

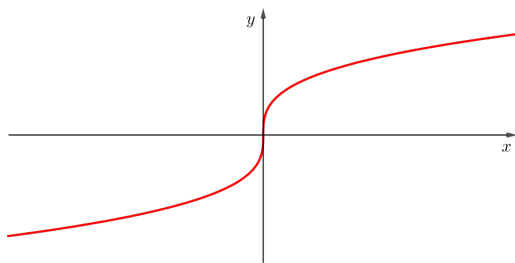
n -tá odmocnina $y = \sqrt[n]{x}$

- $n \in \mathbb{N}; n \geq 2$
- graf: parabola n -tého stupně



n – sudé

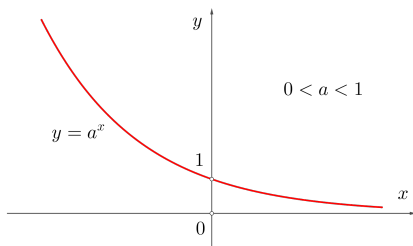
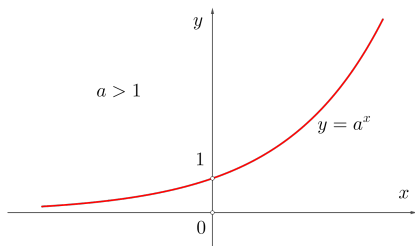
- n sudé, $D(f) = \mathbb{R}_0^+$
- n liché, $D(f) = \mathbb{R}$



n – liché

Exponenciální funkce $y = a^x$

- $a > 0; a \neq 1; a \in \mathbb{R}; D(f) = \mathbb{R}; H(f) = \mathbb{R}^+$



$$\forall x_1, x_2 \in \mathbb{R}$$

- $a^{x_1} \cdot a^{x_2} = a^{x_1+x_2}$

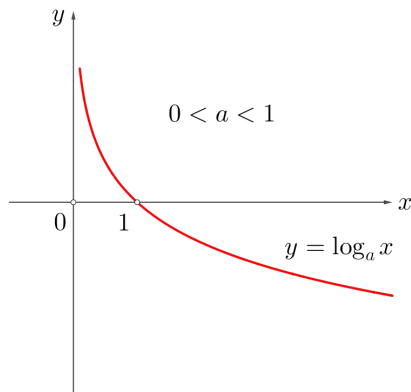
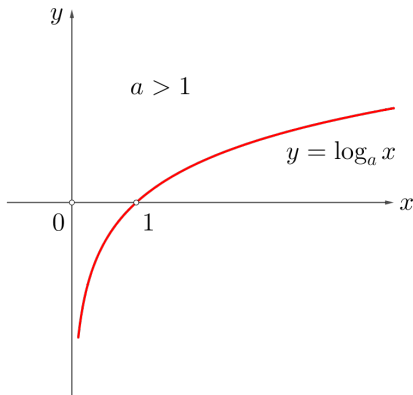
- $\frac{a^{x_1}}{a^{x_2}} = a^{x_1-x_2}$

- $a^x \cdot b^x = (a \cdot b)^x$

- $(a^{x_1})^{x_2} = a^{x_1 \cdot x_2}$

Logaritmická funkce $y = \log_a x$

- $a > 0; a \neq 1; a \in \mathbb{R}; D(f) = \mathbb{R}^+; H(f) = \mathbb{R}$
- **přírozený logaritmus:** $\ln x = \log_e x$, $e \doteq 2,71$
- **dekadický logaritmus:** $\log x = \log_{10} x$

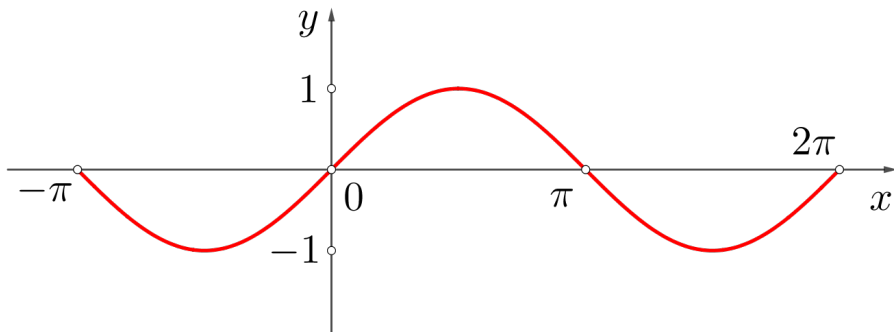


$\forall x_1, x_2 \in \mathbb{R}^+$

- $\log_a x_1 + \log_a x_2 = \log_a (x_1 \cdot x_2)$
- $\log_a x_1 - \log_a x_2 = \log_a \frac{x_1}{x_2}$
- $\log_a x^k = k \cdot \log_a x$

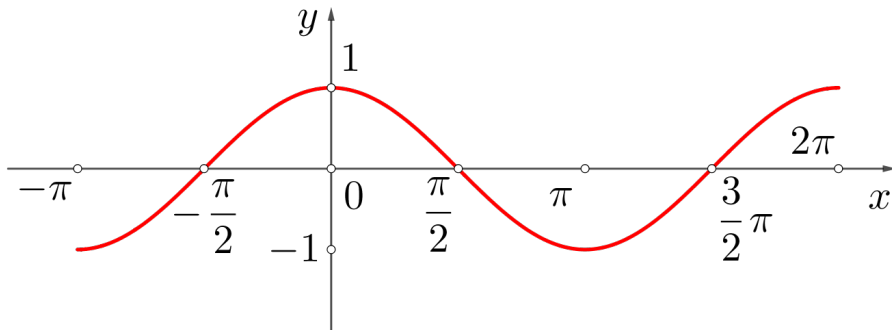
Sinus $y = \sin x$

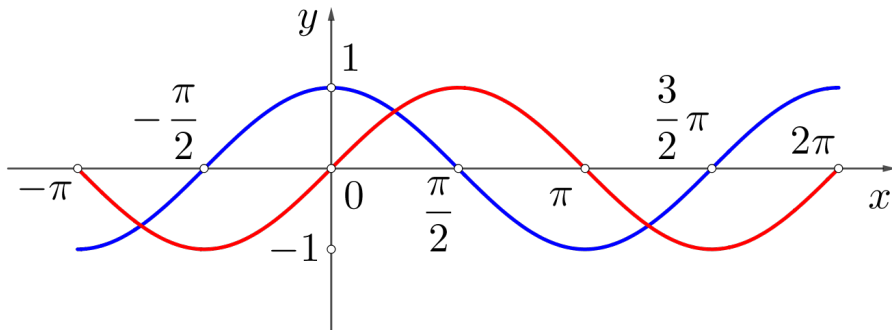
- $D(f) = \mathbb{R}; H(f) = \langle -1, 1 \rangle$
- lichá
- periodická na $\mathbb{R}$ s periodou 2π



Kosinus $y = \cos x$

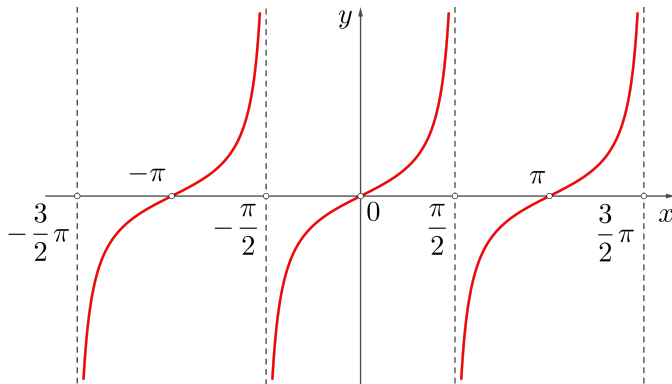
- $D(f) = \mathbb{R}; H(f) = \langle -1, 1 \rangle$
- sudá
- periodická na $\mathbb{R}$ s periodou 2π





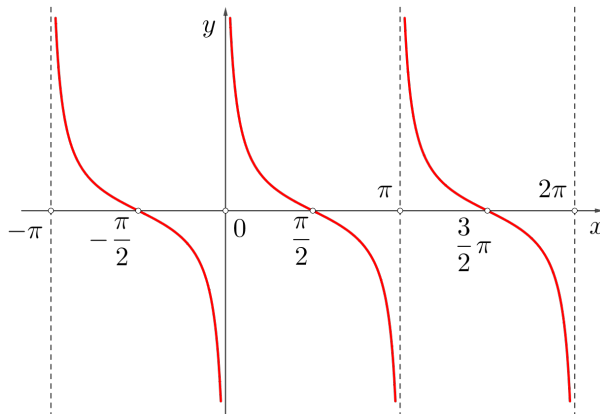
Tangens $y = \operatorname{tg} x = \frac{\sin x}{\cos x}$

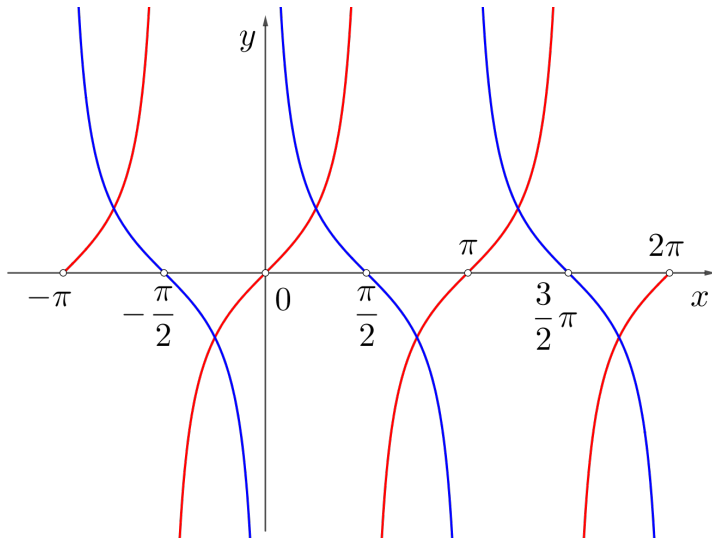
- $D(f) = \mathbb{R} - \left\{ (2k+1)\frac{\pi}{2}; k \in \mathbb{Z} \right\}; H(f) = \mathbb{R}$
- lichá
- periodická na $\mathbb{R}$ s periodou π



Kotangens $y = \cotg x = \frac{\cos x}{\sin x}$

- $D(f) = \mathbb{R} - \{k\pi; k \in \mathbb{Z}\}; H(f) = \mathbb{R}$
- lichá
- periodická na $\mathbb{R}$ s periodou π



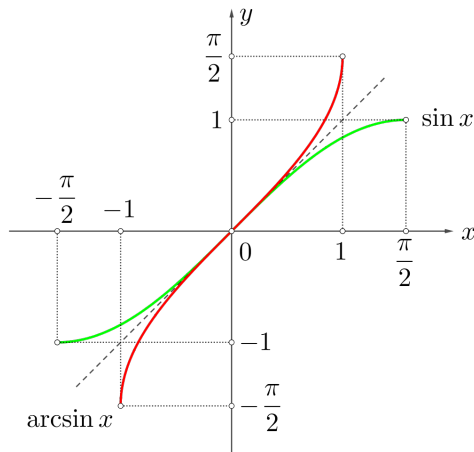


x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\times$	0	$\times$
$\operatorname{cotg} x$	$\times$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\times$	0

- $\sin^2 x + \cos^2 x = 1$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\sin^2 x = \frac{1 - \cos 2x}{2}$
- $\cos^2 x = \frac{1 + \cos 2x}{2}$

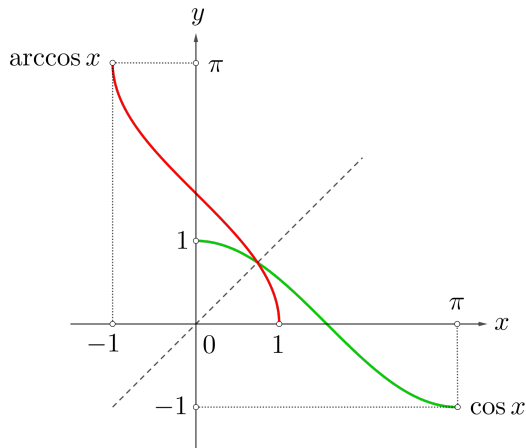
Arkussinus $y = \arcsin x$

■ $D(f) = \langle -1, 1 \rangle; \quad H(f) = \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$



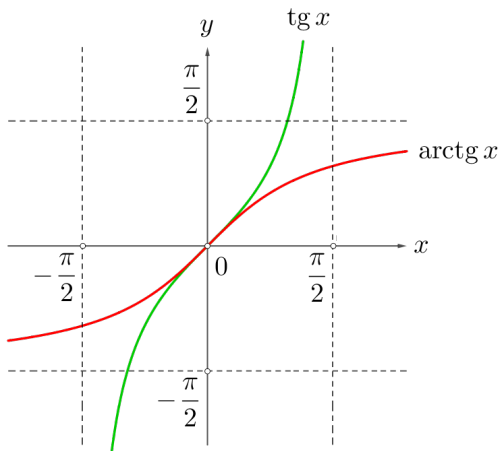
Arkuskosinus $y = \arccos x$

■ $D(f) = \langle -1, 1 \rangle$; $H(f) = \langle 0, \pi \rangle$



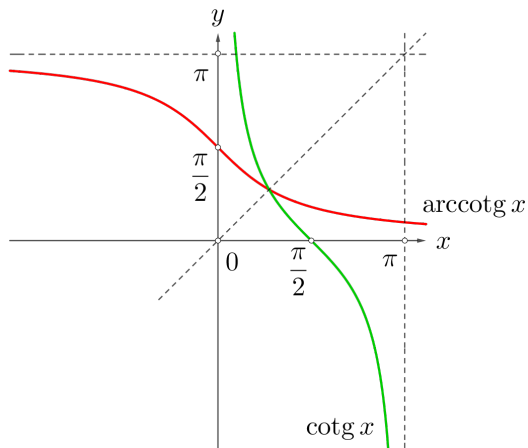
Arkustangens $y = \operatorname{arctg} x$

■ $D(f) = \mathbb{R}; \quad H(f) = \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$



Arkuskotangens $y = \operatorname{arccotg} x$

■ $D(f) = \mathbb{R}; \quad H(f) = \langle 0, \pi \rangle$



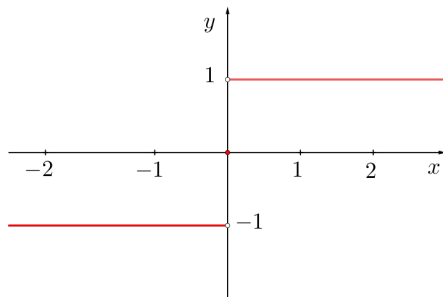
x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arcsin x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\arccos x$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0

x	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{arctg} x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\operatorname{arccotg} x$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

Znaménková funkce $y = \operatorname{sgn} x$

■ $\operatorname{sgn} x : \begin{cases} -1 & \text{pro } x < 0 \\ 0 & \text{pro } x = 0 \\ 1 & \text{pro } x > 0 \end{cases}$

■ lichá



Celá část (dolní celá část) $y = \lfloor x \rfloor$

■ $\lfloor x \rfloor \in \mathbb{Z}, \lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$

