Derivace

11.3.1 Určete derivaci $f'(x)$ a definiční obory $\mathcal{D}(f)$, $\mathcal{D}(f')$ funkcí:

a) $f(x) = \dfrac{4x^7 + 3x^5 - 2x^4 + 7x - 2}{3x^4}$

$$[\ \mathcal{D}(f) = \mathbb{R} - \{0\},\ f'(x) = \frac{12x^7 + 3x^5 - 21x + 8}{3x^5},\ \mathcal{D}(f') = \mathcal{D}(f)\ ]$$

b) $f(x) = (x^3 + 8)(x - 2)$ $\quad [\ \mathcal{D}(f) = \mathbb{R},\ f'(x) = 4x^3 - 6x^2 + 8,\ \mathcal{D}(f') = \mathcal{D}(f)\ ]$

c) $f(x) = \dfrac{e^x - 1}{e^x + 1}$ $\quad [\ \mathcal{D}(f) = \mathbb{R},\ f'(x) = \dfrac{2e^x}{(e^x + 1)^2},\ \mathcal{D}(f') = \mathcal{D}(f)\ ]$

d) $f(x) = \dfrac{1}{\log(3x^2 + x + 1)}$

$$[\ \mathcal{D}(f) = \mathbb{R} - \{0, -\tfrac{1}{3}\},\ f'(x) = -\frac{6x + 1}{(3x^2 + x + 1)\ln 10 \log^2(3x^2 + x + 1)},\ \mathcal{D}(f') = \mathcal{D}(f)\ ]$$

11.3.2 Určete první a druhou derivaci $f'(x)$, $f''(x)$ a příslušné definiční obory funkcí:

a) $f(x) = x\sqrt{x^2 + 3}$

$$[\ f'(x) = \frac{2x^2 + 3}{\sqrt{x^2 + 3}},\ f''(x) = \frac{x(2x^2 + 9)}{\sqrt{(x^2 + 3)^3}},\ \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R}\ ]$$

b) $f(x) = \ln\sqrt{\dfrac{1 - \sin x}{1 + \sin x}}$

$$[\ f'(x) = -\frac{1}{\cos x},\ f''(x) = -\frac{\sin x}{\cos^2 x},\ \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R} - \{\tfrac{\pi}{2} + k\pi, k \in \mathbb{Z}\}\ ]$$

11.3.3 Určete druhou derivaci $f''(x)$ a příslušné definiční obory funkcí:

a) $f(x) = x(\ln x - 1)$ $\quad [\ f''(x) = \dfrac{1}{x},\ \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = (0, \infty)\ ]$

b) $f(x) = \operatorname{arctg}(x - \sqrt{x^2 + 1})$ $\quad [\ f''(x) = -\dfrac{x}{(x^2 + 1)^2},\ \mathcal{D}(f) = \mathcal{D}(f') = \mathcal{D}(f'') = \mathbb{R}\ ]$

11.3.4 Najděte rovnici tečny t a normály n ke grafu funkce $y = f(x)$:

a) $f(x) = e^{-x}\cos 2x$ v bodě $A = [0, ?]$ $\quad [\ t : x + y - 1 = 0,\ n : x - y + 1 = 0\ ]$

b) $f(x) = e^{\frac{x}{2}} + 1$, je-li t rovnoběžná s přímkou $x - 2y + 1 = 0$

$$[\ t : x - 2y + 4 = 0,\ n : 2x + y - 2 = 0\ ]$$